Physical Principles in Biology
Biology 3550
Fall 2018

Lecture 11:

Quiz 1 Recap and

Random Walks Continued

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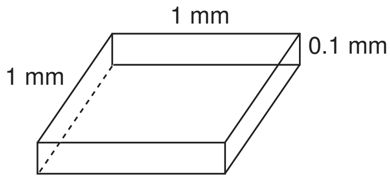
Announcements

- Quiz 2: Wednesday, 19 Sept.
- Problem Set 2 is now posted on course web page.
  Due Monday, 24 Sept.
  Start early!

Quiz 1, Problem 1(a)

- Calculate the volume, in mL, of the chamber shown below:



$$
\begin{aligned}
V &= 1\,\mathrm{mm} \times 1\,\mathrm{mm} \times 0.1\,\mathrm{mm} \\
&= 0.1\,\mathrm{mm}^3
\end{aligned}
$$

- Does $0.1\,\mathrm{mm}^3$ equal 0.1 mL? They both start with “milli”.
- How do we go from mm^3 to mL?

One Way to Get the Result in mL

- Do the calculation in meters, and then convert meters^3 to milliliters.
- Convert dimensions to meters:

$$1\,\text{mm} = 10^{-3}\,\text{m}$$

- Calculate volume in m^3:

$$\begin{aligned} V &= 10^{-3}\,\text{m} \times 10^{-3}\,\text{m} \times 10^{-4}\,\text{m} \\ &= 10^{-10}\,\text{m}^3 \end{aligned}$$

- Convert m^3 to liters

$$1\,\text{m}^3 = 10^3\,\text{L}$$

$$10^{-10}\,\text{m}^3 \times 10^3\,\text{L}/\text{m}^3 = 10^{-7}\,\text{L}$$

- Convert liters to milliliters

$$1\,\text{L} = 10^3\,\text{mL}$$

$$10^{-7}\,\text{L} \times 10^3\,\text{mL}/\text{L} = 10^{-4}\,\text{mL}$$

Another Way to Get the Result in mL

- Do the calculation in centimeters, since 1 cm^3 equals 1 mL.
- Convert dimensions to centimeters:

$$1\,\text{cm} = 10^{-2}\,\text{m} = 10\,\text{mm}$$
$$1\,\text{mm} = 0.1\,\text{cm}$$

- Calculate volume in cm^3:

$$\begin{aligned} V &= 0.1\,\text{cm} \times 0.1\,\text{cm} \times 0.01\,\text{cm} \\ &= 10^{-4}\,\text{cm}^3 \\ &= 10^{-4}\,\text{mL} \end{aligned}$$

Quiz 1, Problem 1(b)

- A human erythrocyte (red blood cell) is roughly disc shaped, with a diameter of about 7 μm and a thickness of about 2 μm.
- What is the volume of an erythrocyte in mL?

Quiz 1, Problem 1(c)

- From the previous parts of the problem:
  - Volume of the chamber: 10^{-4} mL.
  - Volume of an erythrocyte: 7.7×10^{-11} mL.
- How many erythrocytes could fit in the chamber? What assumptions are you making?

Quiz 1, Problem 1(d)

- From part b of the problem:
  - Volume of an erythrocyte: 7.7×10^{-11} mL.
- New information:
  - An adult human has about 20×10^{12} erythrocytes.
- What is the total mass of erythrocytes in an adult human?

Quick Review of Random-Walk Parameters

- n: Number of steps in a single random walk.
- N: Number of random walks used to calculate averages, assumed to be very large.
- x: Coordinate position relative to starting point, $x = 0$.
- x_n: x-coordinate after n steps.
- δ: Step length, always a positive value.
- δ_i: Change in x-coordinate for step number i.
  Can be positive or negative
- $+\delta$ and $-\delta$: Posible values of δ_i, for a step in the positive or negative direction.
- $p_{+\delta}$ and $p_{-\delta}$: Probabilities of steps in the positive or negative direction.

Some Important Averages

- $\langle \delta \rangle$: Mean change in x-coordinate for a single step, over a large number of steps.
- $\langle \delta^2 \rangle$: Mean-square average of displacements for individual steps.
- $\langle x_n \rangle$: Mean value of x_n, over a large number of random walks.
- $\langle x_n^2 \rangle$: Mean-square average of x_n, over a large number of random walks.
- $\mathrm{RMS}(x_n)$: Root-mean-square average of x_n, over a large number of random walks.

Important Results So Far

- The average displacement in a single step:

  $$\langle \delta \rangle = \delta(2p_{+\delta} - 1)$$

- The average value of x_n over a large number of random walks:

  $$\langle x_n \rangle = n\delta(2p_{+\delta} - 1)$$

- The average value of x_n^2 over a large number of random walks:

  $$\langle x_n^2 \rangle = n\langle \delta^2 \rangle$$

  Key assumption: The random walk is unbiased

  $$p_{+\delta} = p_{-\delta} = 0.5$$

  and

  $$\langle x_n \rangle = 0$$

The Mean-square Displacement for One Step: $\langle \delta^2 \rangle$

- The average of δ^2 over a large number of steps is equal to the expected value of δ^2:

$$\begin{aligned}\langle \delta^2 \rangle = E(\delta^2) &= p_{+\delta}(+\delta)^2 + p_{-\delta}(-\delta)^2 \\ &= p_{+\delta}\delta^2 + p_{-\delta}\delta^2 \\ &= \delta^2 \left(p_{+\delta} + p_{-\delta}\right) \\ &= \delta^2\end{aligned}$$

- The mean-square displacement for the random walk:

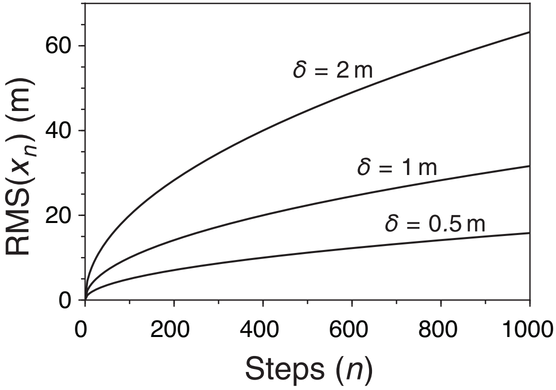
$$\langle x_n^2 \rangle = n\delta^2$$

- The root-mean-square displacement for the random walk:

$$\mathrm{RMS}(x_n) = \sqrt{\langle x_n^2 \rangle} = \sqrt{n}\delta$$

The Root-mean-square Displacement for a One-dimensional Random Walk

$$\mathrm{RMS}(x_n) = \sqrt{\langle x_n^2 \rangle} = \sqrt{n}\delta$$



- This is THE most important thing to remember about random walks!