Physical Principles in Biology
Biology 3550
Spring 2024

# Lecture 14:

# The Gaussian Probability Distribution

Friday, 9 February 2024


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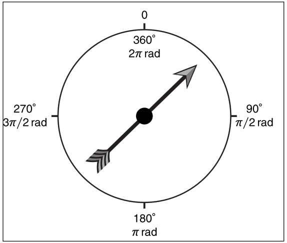
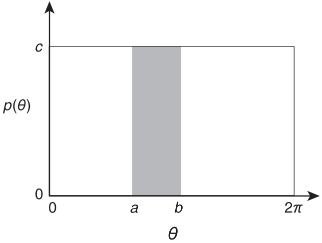
# Announcements

- Problem Set 2:
  - Due 11:59 PM, Monday, 12 February.
  - Download problems from Canvas.
  - Upload work to Gradescope.
  - Show your work!
  - Please don't scrunch things up!
- Quiz 2:
  - Friday, 9 February
  - 25 min, second half of class.

# A Continuous Probability Distribution Function for the Spinner



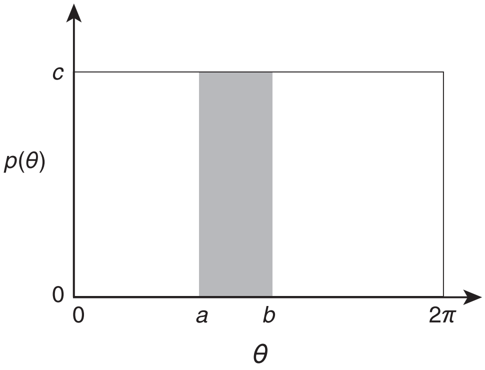




- $\theta$ is a continuous variable, with values from 0 to $2\pi$.
- $p(\theta)$ is a function of $\theta$, with a constant value, $c$, for all values of $\theta$.
- Interpretation of $p(\theta)$: The integral

$$\int_a^b p(\theta) d\theta$$

is the probability that the spinner lands between the values $a$ and $b$.

# An Important Constraint on a Continuous PDF




- To be make sense, the integral over all possible values must equal 1:

$$\int_0^{2\pi} p(\theta) d\theta = 1$$

- Equivalent to the requirement for a discrete PDF that the sum of all probabilities be equal to 1.

- For the spinner pdf, the constant, $c$, is chosen to normalize the PDF.

$$p(\theta) = \frac{1}{2\pi}$$

# The Expected Value (Mean) for a Continuous PDF

- For a discrete random variable, $x$, with discrete PDF, $p(x)$, the expected value is:

$$E(x) = \mu = \sum_{i=1}^{n} p(x_i)x_i$$

- For a continuous random variable, $x$, with range $x_1 \leq x \leq x_2$ and continuous PDF, $p(x)$, the expected value is:

$$E(x) = \mu = \int_{x_1}^{x_2} p(x)x dx$$

- For the spinner variable, $\theta$:

$$E(\theta) = \mu = \int_0^{2\pi} p(\theta)\theta d\theta = \int_0^{2\pi} \frac{1}{2\pi}\theta d\theta = \frac{1}{4\pi}\theta^2 d\theta\Big|_0^{2\pi} = \pi$$

# The Variance for a Continuous PDF

- For a discrete random variable, $x$, with discrete PDF, $p(x)$, the variance is:

$$\sigma^2 = \sum_{i=1}^{n} p(x_i)(x_i - \mu)^2$$

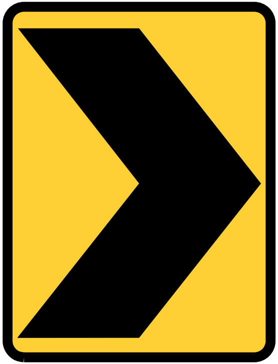
- For a continuous random variable, $x$, with range $x_1 \leq x \leq x_2$ and continuous PDF, $p(x)$, the variance is:

$$\sigma^2 = \int_{x_1}^{x_2} p(x)(x - \mu)^2 dx$$

- For the spinner variable, $\theta$:

$$\sigma^2 = \int_{0}^{2\pi} p(\theta)(\theta - \mu)^2 d\theta = \frac{\pi^2}{3}$$

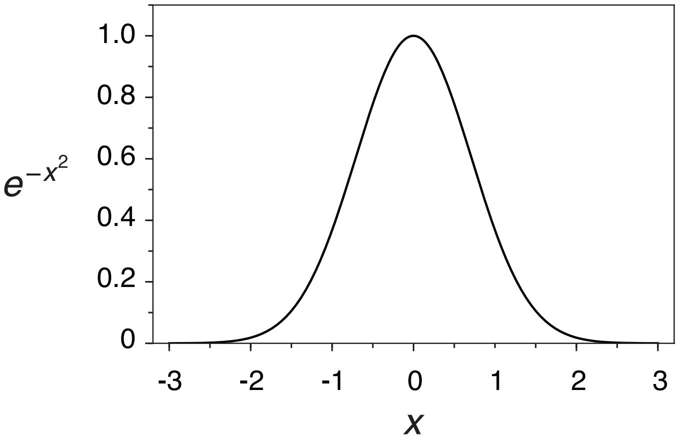
Warning!



Direction Change

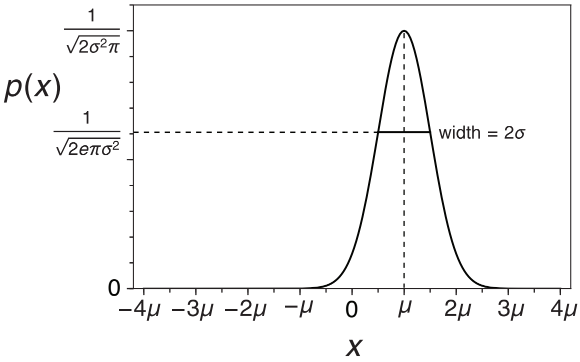
The Gaussian Distribution Function

# The Simplest Form of a Gaussian Function




$$f(x) = e^{-x^2}$$

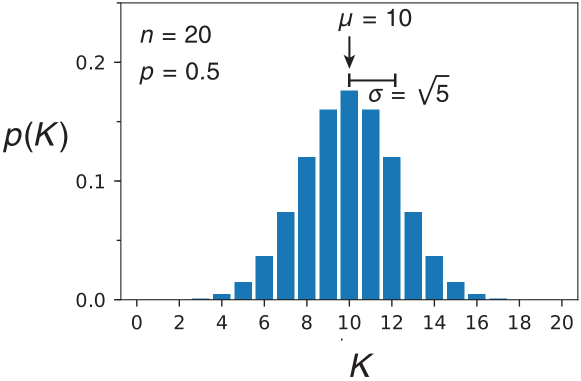
# The Gaussian Probability Distribution Function




$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

- Also called the normal probability distribution function.
- Mean $= \mu$
- Variance $= \sigma^2$
- Standard deviation $= \sigma$

# Mean and Variance for The Binomial Distribution




- Mean

  $$\mu = np_s$$

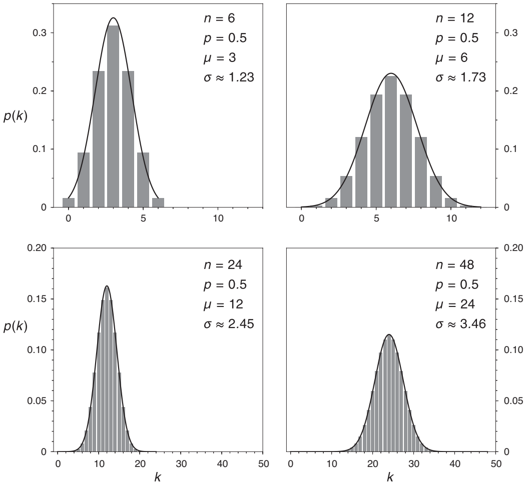
- Variance

  $$\sigma^2 = np_s(1 - p_s)$$

- Gaussian probability function to approximate the binomial distribution function with the same mean and variance:

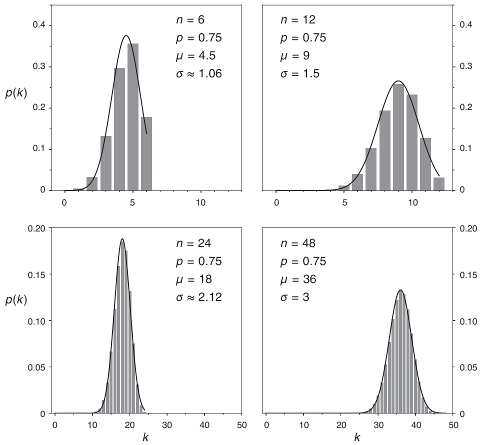
$$p(k) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(k-\mu)^2}{2\sigma^2}} = \frac{1}{\sqrt{2\pi np_s(1 - p_s)}} e^{-\frac{(k-np_s)^2}{2np(1-p_s)}}$$

# Approximation of Binomial Distributions by Gaussian Distributions




- *n* doesn’t have to be very large for a pretty good approximation!

# Approximation of Binomial Distributions by Gaussian Distributions




- It doesn’t work so well if the binomial distribution is biased, with $p_s \neq 0.5$.
- The Gaussian distribution is always symmetrical, but the binomial distribution only is if $p_s = 0.5$.
- If $n$ is large enough, the Gaussian distribution is a good approximation, even if $p_s \neq 0.5$.