Physical Principles in Biology
Biology 3550
Spring 2023

Lecture 19:

Rates of Diffusion

Friday, 23 February 2024

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A Quick Review from the Last Lecture

- Kinetic energy of an object along the x-axis: $E_{\mathrm{k},x} = mv_x^2/2$
- RMS translational kinetic energy of a molecule: $E_{\mathrm{k},x} = kT/2$
- RMS velocity of molecule: $v_x = \delta_x/\tau = \sqrt{kT/m}$
- The diffusion coefficient:

$$D = \frac{\delta_x^2}{2\tau} = \frac{v_x \delta_x}{2}$$

- Calculating δ_x and τ from D and v_x:

$$\delta_x = \frac{2D}{v_x} = \frac{2D}{\sqrt{kT/m}}$$

$$\tau = \frac{\delta_x}{v_x} = \frac{\delta_x^2}{2D}$$

RMS Distance of Diffusion

- Random walk along one direction: $\langle x^2 \rangle = n\delta_x^2$
- For diffusion:

$$D = \frac{\delta_x^2}{2\tau}$$

$$\delta_x^2 = 2D\tau$$

$$n = t/\tau$$

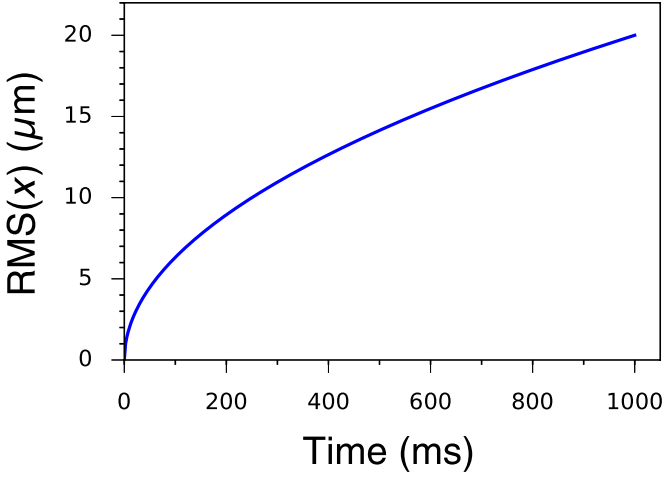
$$\langle x^2 \rangle = n\delta_x^2 = \frac{t}{\tau} 2D\tau = 2Dt$$

$$\mathrm{RMS}(x) = \sqrt{2Dt}$$

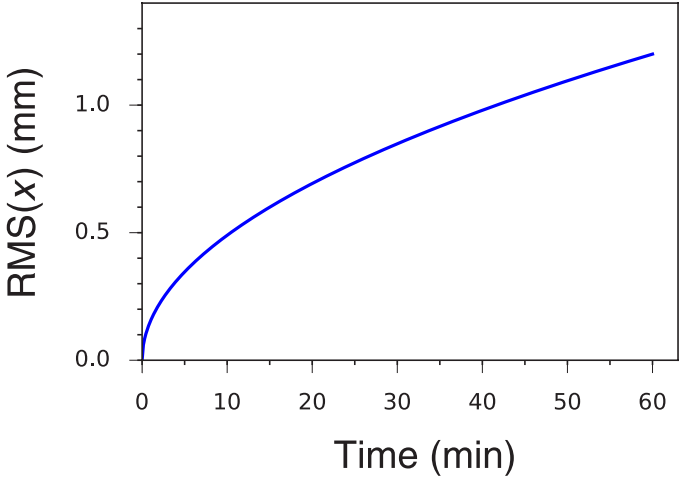
- For bromophenol blue (and molecules of similar size):

$$\mathrm{RMS}(x) = \sqrt{t/s} \times 2 \times 10^{-5}\ \mathrm{m}$$

RMS Distance of Diffusion for a Small Molecule



RMS Distance of Diffusion for a Small Molecule

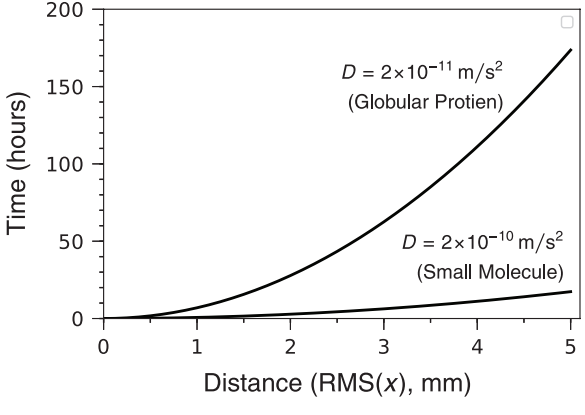


Time Required for Diffusion Over a Range of Distances

$$\text{RMS}(x) = \sqrt{2Dt}$$

$$2Dt = \langle x^2 \rangle$$

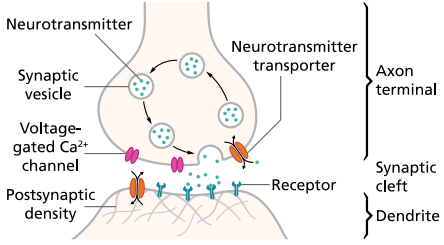
$$t = \langle x^2 \rangle / (2D)$$



- Time required is inversely related to the diffusion coefficient.
- Diffusion is effective over short distances, but not long.

Chemical Communication Between Neurons

- Structure of a synapse



- Synaptic cleft: $\approx 20\,\text{nm}$ wide
- Time for diffusion for a small molecule: $\approx 10^{-6}\,\text{s} = 1\,\mu\text{s}$
- Time to diffuse over the length of a sciatic axon (1 m): $\approx 2 \times 10^{9}\,\text{s} = 80\,\text{yr}$