Physical Principles in Biology
Biology 3550
Fall 2018

Lecture 21:

Diffusion in Gasses and

a Plant Faces Diffusion

Friday, 19 October 2018

©David P. Goldenberg
University of Utah
goldenberg@biology.utah.edu

Some Calculated Diffusion Coefficients

- The Stokes-Einstein equation for spherical particles:

$$D = \frac{kT}{6\pi\eta r}$$

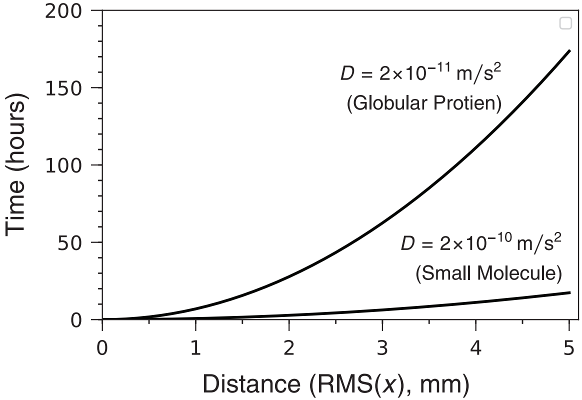
- In water at 25°C:
  - Small molecule (1 nm): 2×10^{-10} m^{2}s^{-1}
  - Protein (10 nm): 2×10^{-11} m^{2}s^{-1}
  - Bacterium (1 μm): 2×10^{-13} m^{2}s^{-1}
  - 1 mm sphere: 2×10^{-16} m^{2}s^{-1}

Time Required for Diffusion Over a Range of Distances

$$\mathrm{RMS}(x) = \sqrt{2Dt}$$

$$2Dt = \langle x^2 \rangle$$

$$t = \langle x^2 \rangle / (2D)$$



- Time required is inversely related to the diffusion coefficient.
- Diffusion is effective over short distances, but not long.

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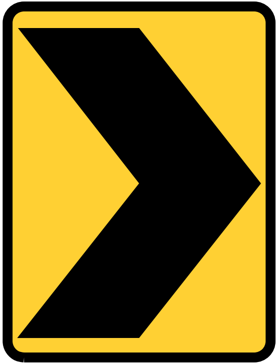


Direction Change

Diffusion of Gasses

Clicker Question #1

How Far Apart are Molecules in Air?
(on average)

A) $\approx 1\,\text{nm}$

B) $\approx 1\,\mu\text{m}$

C) $\approx 1\,\text{mm}$

D) $\approx 1\,\text{cm}$

E) $\approx 1\,\text{m}$

All answers count for now.

Group Problem #1

- Diffusion coefficients of gasses: $\approx 2\times10^{-5}\,\mathrm{m^2/s}$
- From a previous lecture: RMS velocity of N_2: $\approx 300\,\mathrm{m/s}$.
- Calculate the RMS distance an N_2 molecule travels before changing direction.

Diffusion in Gasses

- Diffusion coefficients of gasses: $\approx 2\times10^{-5}\,\mathrm{m^2/s}$
- From a previous lecture: RMS velocity of N_2: $\approx 300\,\mathrm{m/s}$.
- $D = \delta_x^2/(2\tau)$, and $v = \delta_x/\tau$

  $$\delta_x = 2D/v = 1.3\times10^{-7}\,\mathrm{m}$$

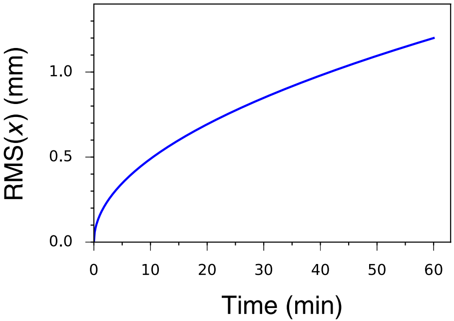
  The “mean-free-path” distance.
  Is this the distance between molecules?
- Time between collisions:

  $$\tau = \delta_x/v \approx 4.4\times10^{-10}\,\mathrm{s}$$

- Much longer random walk steps than in liquids, but still pretty short.

Diffusion in Liquids vs. Atmosphere

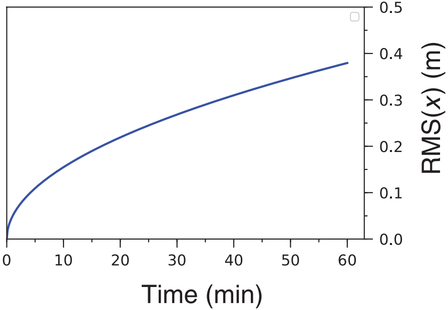
A small molecule in water,
$D = 2\times10^{-10}\ \mathrm{m^2/s}$



$$\delta_x = 6.5\times10^{-12}\ \mathrm{m}$$

$$\tau = 10^{-13}\ \mathrm{s}$$

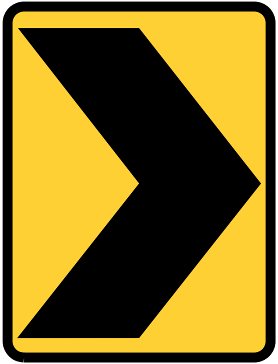
N_2 in atmosphere,
$D = 2\times10^{-5}\ \mathrm{m^2/s}$



$$\delta_x = 1.3\times10^{-7}\ \mathrm{m}$$

$$\tau = 4.4\times10^{-10}\ \mathrm{s}$$

Warning!



Direction Change

A Plant Faces Diffusion

Growth of a Hypothetical Plant

- 1 kg carbon per year, for net growth and replacement of leaves.
- Where does the carbon come from?

  From thin air!

  $$6CO_2 + 6H_2O \longrightarrow C_6H_{12}O_6 + 6\,O_2$$

  An extremely unfavorable reaction, unless coupled to the energy of absorbed light.

Group Problem #2

How much CO_2 (in moles) does the plant have to assimilate per second?

(for 1 kg carbon/yr)

Growth of a Hypothetical Plant

- How much CO_2 (in moles) does the plant have to assimilate per second?

$$1\,\text{kg} \div 12\,\text{g/mol} \approx 80\,\text{mol}$$

$$1\,\text{yr} \times 365\,\text{days/yr} \times 24\,\text{hr/day} \times 60\,\text{min/hr} \times 60\,\text{s/min} \approx 3{\times}10^{7}\,\text{s}$$

- But, CO_2 is incorporated only during daylight, so the total time available is only about half of this.

$$80\,\text{mol} \div 1.5{\times}10^{7}\,\text{s} \approx 5{\times}10^{-6}\,\text{mol/s}$$

Growth of a Hypothetical Plant

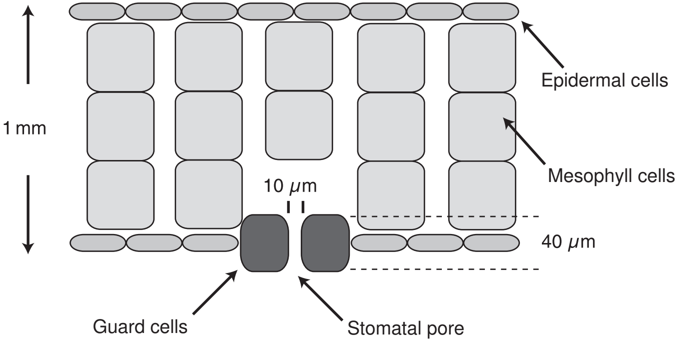
- Total rate of CO_2 assimilation: 5×10^{-6} mol/s
- Total leaf area: $\approx 0.1\,\mathrm{m}^2$
- Flux, per second, per unit of leaf area:

$$J = 5\times10^{-6}\,\mathrm{mol/s} \div 0.1\,\mathrm{m}^2$$

$$= 5\times10^{-5}\,\mathrm{mol}\cdot\mathrm{s}^{-1}\mathrm{m}^{-2}$$

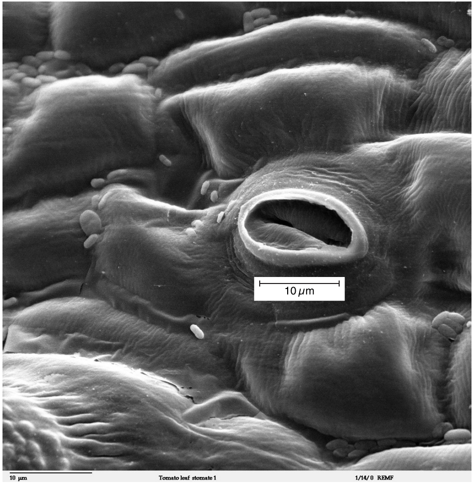
- But, diffusion does not take place across all of the leaf area.

Cross-section of a Plant Leaf



- Photosynthesis takes place in chloroplasts of mesophyll cells.
- Stomata control diffusion of gasses into and out of leaves.
- Diffusion through the stomata takes place in gas phase.

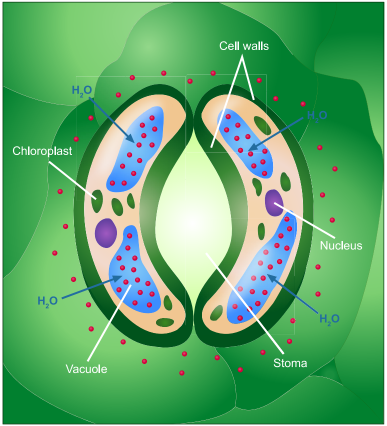
Scanning Electron Micrograph of a Tomato leaf stoma



Micrograph by Louisa Howard.
http://remf.dartmouth.edu/images/botanicalLeafSEM/source/16.html

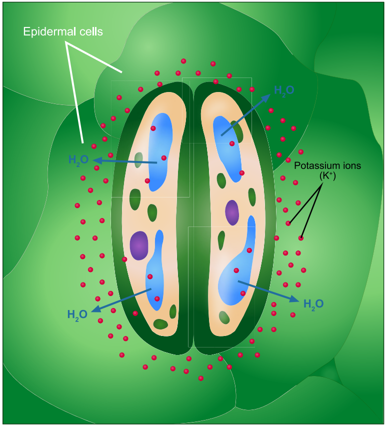
Opening and Closing of Stomata

Guard cells (swollen)



Stoma opening

Guard cells (shrunken)

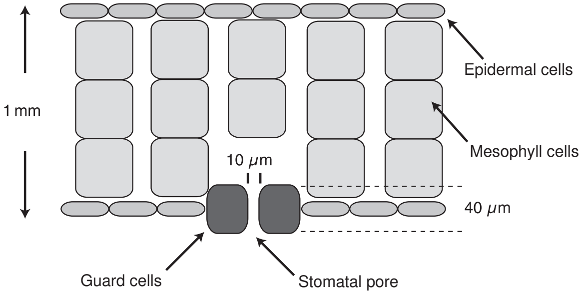


Stoma closing

Illustration by Ali Zifan

https://en.wikipedia.org/wiki/Stoma

Cross-section of a Plant Leaf



- CO_2 diffuses through stomata into leaf airspace.
- CO_2 diffuses into mesophyll cells and then into chloroplasts.
- CO_2 is reduced, or “fixed”, into sugars by ribulose-1,5-bisphosphate carboxylase (Rubisco).
- Steady-state concentration of CO_2 in airspace is about 1/2 atmospheric concentration.

Group Problem #3

- Diffusion coefficient of CO_2 at atmospheric pressure and 298 K: $D = 1.5 \times 10^{-5}\,\mathrm{m}^2/\mathrm{s}$.
- Atmospheric CO_2 concentration: $15\,\mu\mathrm{M}$ (≈ 400 ppm)
- CO_2 concentration in leaf airspace: $7.5\,\mu\mathrm{M}$
- Length of stomatal pore: $\approx 40\,\mu\mathrm{m}$
- Calculate the flux, J, of CO_2 through the stomata.

Diffusion of CO_2

- Diffusion coefficient of CO_2 at atmospheric pressure and 298 K:

  $D = 1.5 \times 10^{-5}\ \mathrm{m^2/s}$.

- Concentration gradient:
  - Atmospheric CO_2 concentration: $15\,\mu\mathrm{M} = 1.5 \times 10^{-2}\ \mathrm{mol/m^3}$. ($\approx$ 400 ppm)
  - CO_2 concentration in leaf airspace: $7.5\,\mu\mathrm{M} = 7.5 \times 10^{-3}\ \mathrm{mol/m^3}$
  - Length of stomatal pore: $\approx 40\,\mu\mathrm{m} = 4 \times 10^{-5}\ \mathrm{m}$

$$\frac{dC}{dx} \approx \frac{7.5 \times 10^{-3}\ \mathrm{mol/m^3}}{4 \times 10^{-5}\ \mathrm{m}} = 175\ \mathrm{mol \cdot m^{-4}}$$

- Flux:

$$J = -D\frac{dC}{dx} = -1.5 \times 10^{-5}\ \mathrm{m^2/s} \times 175\ \mathrm{mol \cdot m^{-4}}$$

$$= -2.6 \times 10^{-3}\ \mathrm{mol \cdot m^{-2} s^{-1}}$$