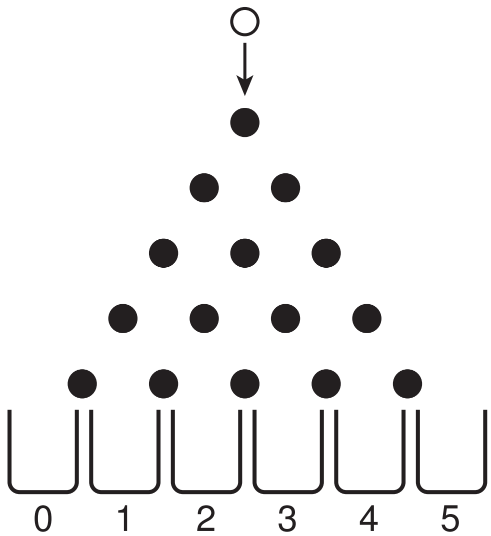
Name:________________________________

Biology 3550
Physical Principles in Biology
Fall Semester 2017

Final Exam
12 December 2017

100 points total
Be sure to show your work, convert your answers to decimal form and include correct units in all of your answers!

1. Consider the five-row plinko illustrated below:



(a) (10 pts) In the table below, enter the number of paths to each of the 6 buckets and the probability of a ball falling in each bucket. Assume that the probabilities of the ball falling left or right are equal at each peg. Space is provided on the next page for calculations.

Bucket No.	Paths	Probability
0		
1		
2		
3		
4		
5		

(b) (5 pts) Suppose that someone has modified just two pegs: The first and last pegs on the bottom row. Each of these pegs has been altered so that the probability of the ball falling to the left is 0.75. Calculate the new probabilities of a ball falling into buckets 0 and 5.

(c) (5 pts) For the modified plinko, calculate the probabilities of a ball falling into buckets 1, 2, 3 and 4.

2. A population of bacteria are swimming (independently) through a liquid with a velocity of $10\,\mu$m/s. Each bacterium randomly changes directions every 1 s, on average.

   (a) (5 pts) What is RMS distance between the starting and ending positions after the bacteria swim for 20 min.

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(b) (5 pts) Suppose that the bacteria were to increase their velocity to 20 μm/s, while still changing direction every 1 s. What would the RMS end-to-end distance be after 20 min?

(c) (5 pts) Suppose that the bacteria were to increase the time between changing directions, to 2 s, while keeping the velocity at 10 μm/s. What would the RMS end-to-end distance be after 20 min?

3. As we discussed in class, some bacteria, including *Escherichia coil*, are able to improve their odds in the random walk of life by adjusting their swimming behaviour when they detect nutrients.

   (a) (5 pts) Briefly describe the strategy that *E. coli* bacteria use to find food.

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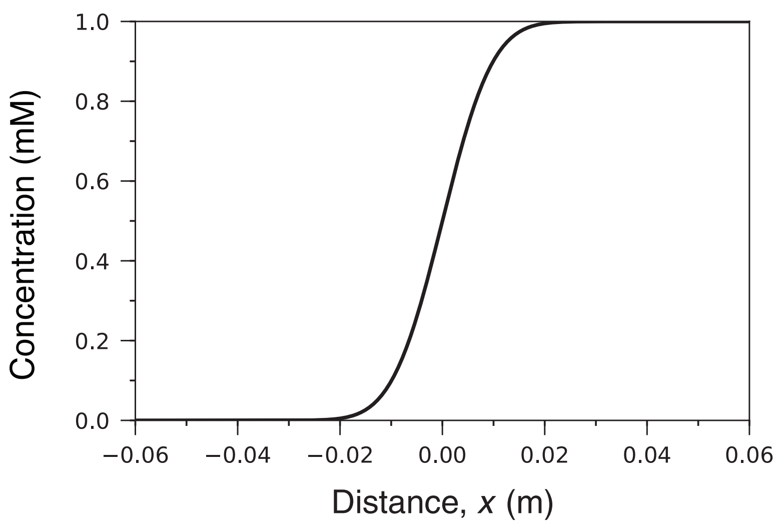
(b) (5 pts) Suppose that an *E. coli* bacterium is swimming chemotactically, with a velocity of 10 μm/s, and changes direction every 1 s *on average*. How would the RMS end-to-end distance, over 20 min, change from what it would be if it were swimming completely randomly at the same velocity and with the same average time between direction changes? You do not need to do any calculations for this problem, but you should briefly explain your answer.

4. The graph below represents the diffusion of molecules from a sharp boundary, originally located at position 0 on the x-axis. The molecules have been diffusing for about 24 h.



(a) (5 pts) To the graph above, add a curve representing concentration as a function of distance after the molecules have been allowed to diffuse for another 24 h.

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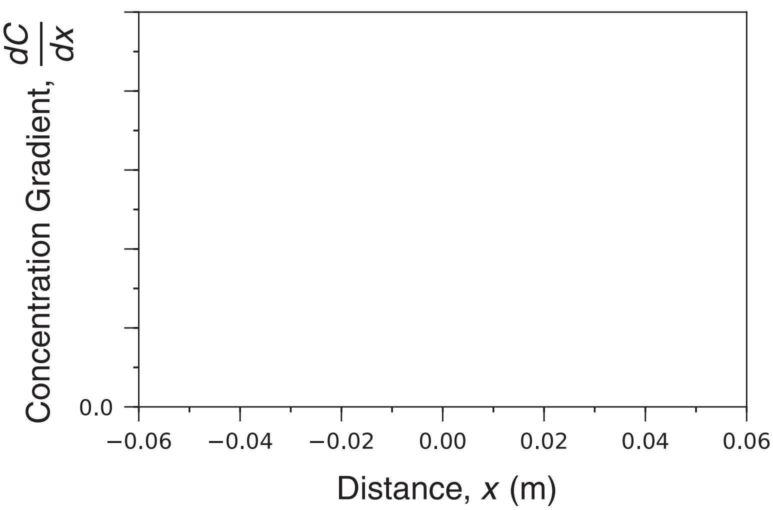
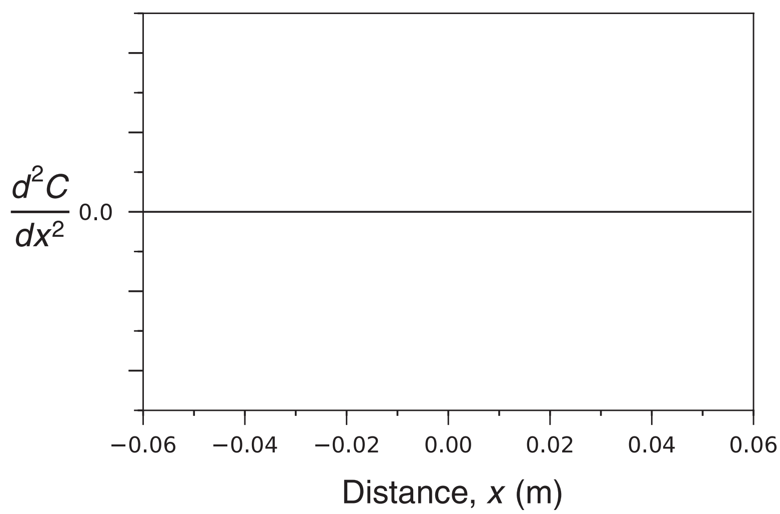
(b) (5 pts) From the original graph, calculate the concentration gradient at $x = 0$. Express your result in base SI units.

(c) (5 pts) Using the axes below, make graphs of the concentration gradient versus x, for both 24 h and 48 h after the beginning of the experiment. Add the correct units to the vertical axis, but you do not need to add numerical values. Be sure to label the two curves.



(d) (5 pts) Briefly explain the significance of the concentration gradient. In particular, what important quantity is determined by dC/dx, and what equation defines this relationship?

(e) (5 pts) Using the axes below, make a graph of the second derivative of concentration with respect to x, for 24 h after the beginning of the experiment. Add the correct units to the vertical axis, but you do not need to add numerical values.



(f) (5 pts) Briefly explain the significance of the second derivative of concentration with respect to x. In particular, what important quantity is determined by d^2C/dx^2, and equation defines this relationship?

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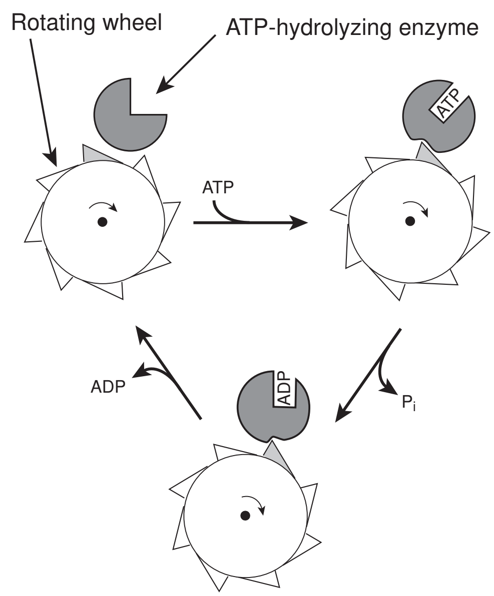
5. As a means of introducing some of the basic principles of molecular motors, we discussed in class the hypothetical ATP-powered motor illustrated in the diagram below.



(a) (6 pts) Describe two important features that this hypothetical motor illustrates and are found in the real ATP-driven motors that we discussed in class.

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(b) (8 pts) Suppose that a motor of this design were functioning in a cell in which the concentrations of ATP, ADP and inorganic phosphate were $1\,\mu$M, $100\,$mM and $10\,$mM, respectively, and the temperature is 25°C. Under these conditions, what is the maximum amount of work that could be obtained from the hydrolysis of ATP *during one revolution of a single motor*?

(c) (4 pts) Under the conditions described above, would the motor run clockwise or counter-clockwise? Briefly explain your answer.

(d) (7 pts) assume the following about the motor:

- The motor uses ATP with an efficiency of 50%.
- The weel of the motor has a radius of 10 nm
- The force exerted on the wheel during each revolution is approximately constant.

What is the average force exerted by the ATPase on the teeth of the wheel?

Happy holidays and best wishes for the new year!

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Possibly Useful Constants and Equations

Avogadro's number, $N_{\mathrm{A}} = 6.02\times10^{23}$
Gas constant: $R = 8.314\,\mathrm{L\cdot kPa\cdot K^{-1}mol^{-1}} = 8.314\,\mathrm{J\cdot K^{-1}mol^{-1}}$
Boltzmann constant: $k = 1.38\times10^{-23}\,\mathrm{J\cdot K^{-1}}$
The standard free energy change for ATP hydrolysis: -30 kJ/mol

$1\,\mathrm{N} = 1\,\mathrm{kg\cdot m\cdot s^{-2}}$

$1\,\mathrm{J} = 1\,\mathrm{N\cdot m} = 1\,\mathrm{kg\cdot m^2\cdot s^{-2}}$

$1\,\mathrm{L} = 10^{-3}\,\mathrm{m^3}$

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}}e^{-(x-\mu)^2/(2\sigma^2)}$$

$$\langle r^2\rangle = n\delta^2$$

$$\mathrm{RMS}(r) = \sqrt{\langle r^2\rangle}$$

$$\langle x^2\rangle = n\delta^2/2$$

$$\langle x^2\rangle = n\delta^2/3$$

$$D = \frac{\delta_x^2}{2\tau}$$

$$J = -D\frac{dC}{dx}$$

$$\frac{dC}{dt} = D\frac{d^2C}{dx^2}$$

$$E_{\mathrm{k},x} = mv^2/2$$

$$\mathrm{RMS}(E_{\mathrm{k},x}) = kT/2$$

$$\mathrm{RMS}(v) = \sqrt{kT/m}$$

$$D = \frac{kT}{6\pi\eta r}$$

$$\Delta E = q + w$$

$$\Delta S_{\mathrm{sys}} = \frac{q_{\mathrm{rev}}}{T}$$

$$\Delta S_{\mathrm{surr}} = -\frac{q}{T}$$

$$S_{\mathrm{sys}} = k\ln\Omega$$

$$\Delta S_{\mathrm{sys}} = nR\ln\frac{V_2}{V_1}$$

$$\Delta H = q_{\mathrm{p}}$$

$$\Delta F = \Delta E - T\Delta S_{\mathrm{sys}}$$

$$\Delta G = \Delta H - T\Delta S_{\mathrm{sys}}$$

$$\Delta G = \Delta F - w_{\mathrm{p}}$$

$$\Delta G^\circ = -RT\ln K_{\mathrm{eq}}$$

$$\Delta G = \Delta G^\circ + RT\ln Q$$