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Biology 3550
Physical Principles in Biology
Fall Semester 2016

Mid-Term Exam
7 October 2016

100 points total
Please write your name on each page.
Be sure to show your work and include correct units in all of your answers!
Numerical answers should be evaluated (not left in the form of an equation).

1. (10) pts) Some types of animal cells are connected to their neighbors via structures known as gap junctions. These structures are formed by the close juxtaposition of the plasma membranes of adjacent cells, and the two cells are chemically linked via pores, called connexons. Gap junctions have a variety of specific functions in different cell types and tissues, but their general function appears to be to allow small molecules and ions to move rapidly between cells, thereby establishing a shared metabolic state.

   Suppose that two cells are connected by a gap junction, and that the concentration of a hormone is initially $10\,\mu$M in both cells, but it rapidly increases to $15\,\mu$M in one one of them. Each of the cells is roughly cubic in shape, with edges $30\,\mu$m long. How many *net* molecules must move from the high-concentration cell to the other cell in order to equalize the concentrations?

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2. Although six-sided dice are the most common, dice with other numbers of sides have been devised and are used in some games. Suppose that you have a five-sided die, marked with the numbers 1 through 5, and that the die is fair, that is the probabilities of each of the numbers showing is equal. In this game the result is determined by whether or not the number showing is odd or even.

   (a) (5 pts) Calculate the probabilities of an even (p_e) or odd (p_o) number for a single throw of the die.

   (b) (5 pts) A game is defined as eight successive throws of the die, and the outcome of a game is defined as the specific sequence of even and odd numbers; for instance the sequence [odd, odd, even, odd, odd, even, even, even].
   How many possible sequences are there?

   (c) (8 pts) What sequence has the highest probability? Calculate the probability of this sequence.

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(d) (8 pts) Calculate the probability of seeing exactly five odd numbers in eight throws of the die.

3. Many species of bacteria (including a common resident of the human gut, *Escherichia coli*) can swim through liquid media, and they take three-dimensional random walks[1]. A bacteriologist has been following the swimming of bacteria under specific conditions and has found that these particular bacteria swim in nearly straight lines for 5 s at a time and then randomly change directions and swim in a new direction for 5 s. She has also determined that the swimming velocity (in three dimensions) is 10 μm/s.

(a) (5 pts) What is the total distance ("pedometer distance", if bacteria had swimming pedometers), in mm, that a bacterium will swim in 1 hr?

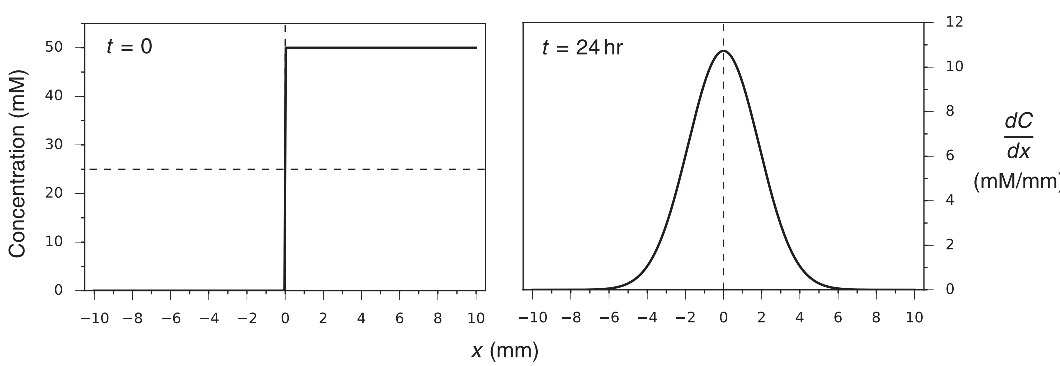
(b) (5 pts) ow many times will a bacterium change directions in 1 hr?

[1]We will return to this subject shortly after fall break.

(c) (8 pts) For a large number of bacteria following this behavior, calculate the RMS distance (in mm) from the starting position to the end point, after 1 hr.

(d) (8 pts) If the bacteria were to increase the duration of their straight swimming segments to 10 s, what would the RMS end-to-end distance (in mm) be after 1 hr?

4. The figures below show simulated results for an experiment monitoring diffusion from a sharp boundary. The graph on the left shows the concentration profile at the beginning of the experiment, whereas the graph on right is a plot of the derivative of concentration with respect to x 24 hr later. The diffusion coefficient for the molecule is 2×10^{-11} m^2/s.



(a) (6 pts) On the left-hand graph, sketch in the concentration profile that would be expected at 24 hr.

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(b) (8 pts) For $x = 0$ and $t = 24\,\text{hr}$, calculate the flux, J (in units of $\text{mol}/(\text{m}^2\text{s})$).

(c) (8 pts) For $x = 0$ and $t = 24\,\text{hr}$, calculate the rate of change in concentration (in units of mM/s).

(d) (8 pts) From the graphs on the previous page, estimate the value of x at which the concentration increases most rapidly, when $t = 24\,\text{hr}$. Explain how you chose this point, and mark and label it on the right-hand graph.

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(e) (8 pts) Calculate the rate of change in concentration (in units of mM/s) at the point where the concentration changes most rapidly at $t = 24\,\text{hr}$.

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(Possibly) Useful Equations and Constants

$$1\,\text{N} = 1\,\text{kg}\cdot\text{m}\cdot\text{s}^{-2}$$

$$1\,\text{J} = 1\,\text{N}\cdot\text{m} = 1\,\text{kg}\cdot\text{m}^2\cdot\text{s}^{-2}$$

$$1\,\text{L} = 10^{-3}\,\text{m}^3$$

$$\binom{n}{k} = \frac{n!}{k!(n-k!)}$$

$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}}e^{-(x-\mu)^2/(2\sigma^2)}$$

$$\langle r^2\rangle = n\delta^2$$

$$\text{RMS}(r) = \sqrt{\langle r^2\rangle}$$

$$\langle x^2\rangle = n\delta^2/2$$

$$\langle x^2\rangle = n\delta^2/3$$

$$D = \frac{\delta_x^2}{2\tau}$$

$$J = -D\frac{dC}{dx}$$

$$\frac{dC}{dt} = D\frac{d^2C}{dx^2}$$

$$E_{\text{k},x} = mv^2/2$$

$$\text{RMS}(E_{\text{k},x}) = kT/2$$

$$D = \frac{kT}{6\pi\eta r}$$

Avogadro's number, $N_\text{A} = 6.02{\times}10^{23}$
Gas constant: $R = 8.134\,\text{L}\cdot\text{kPa}\cdot\text{K}^{-1}\text{mol}^{-1} = 8.134\,\text{J}\cdot\text{K}^{-1}\text{mol}^{-1}$
Boltzmann constant: $k = 1.38{\times}10^{-23}\,\text{J}\cdot\text{K}^{-1}$