Name:________________________________

Biology 3550

Physical Principles in Biology

Fall Semester 2017

Mid-Term Exam

6 October 2017

100 points total

Please write your name on each page.

In your answers you should:

- Show your work or provide an explanation for an answer.
- Use correct units and the correct number of significant figures (One extra significant figure is allowable.)
- Express numerical answers as decimal values, using scientific notation for numbers outside of the range -0.01 to 0.01.

1. Two students are exploring probability by tossing coins. Each experiment consists of tossing a coin ten times, and the result is defined in terms of the sequence of heads and tails for the individual tosses.

   (a) (5 pts) How many possible outcomes (sequences of heads and tails) are there for a single experiment (10 coin tosses)?

   (b) (5 pts) Assuming that the coin is fair, which of the possible outcomes is most likely? What is the probability of this outcome?

Name:________________________________

(c) (5 pts) Suppose that the coin was, in fact, biased, so that the probability of heads for any individual toss is 0.37. Which of the possible outcomes is the most likely? What is the probability of this outcome?

(d) (10 pts) Assuming, again, that the probability of heads for a single toss is 0.37, what is the probability of observing six heads in a single experiment?

Name:______________________

2. A famous entomologist, Prof. Shirley Bugsmee, is studying the behavior of ants of a particular species. She places an ant at the middle of a grid that is completely isolated from any food or other substances the ants can detect, and then watches its behavior. It appears to her that the ant is taking a random walk in which it moves forward some distance, changes direction randomly and then repeats itself. She has recruited several students to help in her experiments, and together they follow the behavior of 100 ants, each for a total of 50 min. They find that the average time between changes in direction is 5 s, and the RMS average distance the ants move away from their starting position in 50 min is 0.25 m.

   (a) (10 pts) Assuming that the ants really are taking a simple random walk, what is the RMS average distance they move between changes in direction?

   (b) (10 pts) Suppose the ants were allowed to walk for 150 min. What would you expect their RMS average distance from the starting position to be?

Name:______________________________

3. Many animal cells are connected to their neighbors via structures known as gap junctions. These structures are formed by the close juxtaposition of the plasma membranes of adjacent cells, and the two cells are chemically linked via pores, called connexons, as illustrated in the figure below (Taken from The Molecular Biology of the Cell, by Alberts *et al.*):

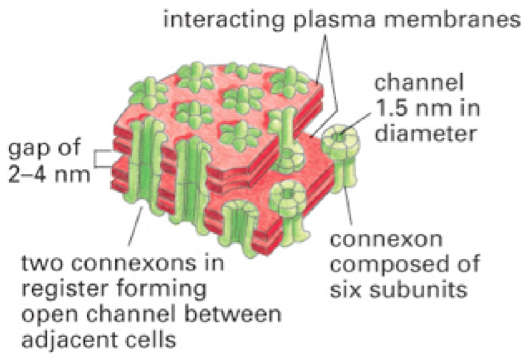

Figure 19–15. Molecular Biology of the Cell, 4th Edition.

The number of connexons in a gap junction can vary greatly, from just a few to several thousand. The pore formed by each connexon has a diameter of 1.5 nm and a length of 10 nm. The junctions allow small molecules and ions to move rapidly between cells, thereby establishing a partially shared metabolic state.

Suppose that two cells are connected by a gap junction containing 200 connexons. A hormone, initially absent in both cells is suddenly synthesized in one of the cells, raising the concentration in that cell to 20 μM. Assume for the following that the diffusion coefficient for the hormone is $5\times10^{-11}\,\mathrm{m^2s^{-1}}$.

(a) (10 pts) Immediately after the concentration change in one of the cells, what is the concentration gradient (in basic SI units) across one of the connexons?

(b) (10 pts) What will be the initial flux, in basic SI units, through the connexons?

(c) (15 pts) Calculate the total net number of hormone molecules that will diffuse across the entire gap junction in 0.01 s, assuming that the concentrations do not change significantly in that time.

Name:______________________________

4. The hormone described in the previous problem has a molar mass of 1,500 g/mol and the temperature of the organism is 37°C.

   (a) (10 pts) Calculate the translational kinetic energy along a single direction, in joules, of the hormone.

   (b) (10 pts) As the molecules diffuse through a solution via a random walk, the average time interval between direction changes is 3×10^{-13} s. What is the RMS average distance the molecules move between direction changes?

Name:________________________________

(Possibly) Useful Equations and Constants

$$1\,\mathrm{N} = 1\,\mathrm{kg}\cdot\mathrm{m}\cdot\mathrm{s}^{-2}$$

$$1\,\mathrm{J} = 1\,\mathrm{N}\cdot\mathrm{m} = 1\,\mathrm{kg}\cdot\mathrm{m}^2\cdot\mathrm{s}^{-2}$$

$$1\,\mathrm{L} = 10^{-3}\,\mathrm{m}^3$$

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-\mu)^2/(2\sigma^2)}$$

$$\langle r^2 \rangle = n\delta^2$$

$$\mathrm{RMS}(r) = \sqrt{\langle r^2 \rangle}$$

$$\langle x^2 \rangle = n\delta^2/2$$

$$\langle x^2 \rangle = n\delta^2/3$$

$$D = \frac{\delta_x^2}{2\tau}$$

$$J = -D\frac{dC}{dx}$$

$$\frac{dC}{dt} = D\frac{d^2C}{dx^2}$$

$$E_{\mathrm{k},x} = mv^2/2$$

$$\mathrm{RMS}(E_{\mathrm{k},x}) = kT/2$$

$$D = \frac{kT}{6\pi\eta r}$$

Avogadro's number, $N_\mathrm{A} = 6.02\times10^{23}$
Gas constant: $R = 8.134\,\mathrm{L}\cdot\mathrm{kPa}\cdot\mathrm{K}^{-1}\mathrm{mol}^{-1} = 8.134\,\mathrm{J}\cdot\mathrm{K}^{-1}\mathrm{mol}^{-1}$
Boltzmann constant: $k = 1.38\times10^{-23}\,\mathrm{J}\cdot\mathrm{K}^{-1}$