Physical Principles in Biology
Biology 3550
Spring 2024

Lecture 2:

Introduction to Measurement and Units

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Clicker Question #1:

What time does class start?

A) 9:30 AM

B) 9:40 AM

C) 9:45 AM

D) 9:50 AM

E) What class?

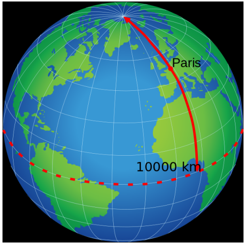
A Classic Movie About the Range of Length Scales

- Created by Charles and Ray Eames in 1977 for IBM
- http://www.eamesoffice.com/the-work/powers-of-ten
- https://www.youtube.com/watch?v=0fKBhvDjuy0

Original Metric System – France, 1795

Defined two basic units:

- meter (*mètre*): One ten-millionth of the distance from the Equator to the North Pole along the meridian passing through Paris.



- gram (*gramme*): Mass of one cm^3 of pure water at the melting temperature of ice.

https://en.wikipedia.org/wiki/Metric_system

Original Reference Objects for Meter and Kilogram

- *Métre des Archives* (1799)
  - Platinum bar based on measurement of the meridian and placed in the French National Archives.
  - Actually short of the definition based on the meridian, by 0.2 mm, but used anyway.
  - Subsequently replaced by other metal bars.
  - Eventually replaced by a definition based on the speed of light: Distance travelled by light in vacuum in 1/299,792,458 of a second.
- *Kilogramme des Archives* (1799)
  - Cylinder of platinum based on the mass of water and placed in the French National Archives.
  - Replaced in 1889 by the international prototype kilogram (IPK), made of platinum-iridium alloy.
  - IPK was the standard definition of the kilogram until 2019!

International Prototypes for kg and meter (British versions)

Photograph from http://www.metric.org.uk

Another Way to Establish a Standard for Mass

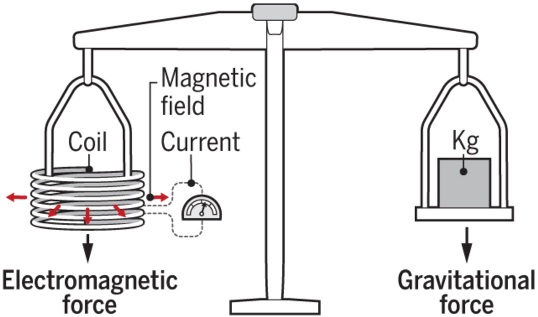


Illustration from Cho, A. (2017). *Science*, 356, 670–671.
http://dx.doi.org/10.1126/science.356.6339.670

- One of the ways used to redefine the kg in 2019, by reference to a defined physical constant, Planck's constant.

- Kibble balance, or "moving coil Watt balance"
- Electric current in coil, within a magnetic field, generates a force opposite to gravitational force on the other side.
- Current is adjusted to match the gravitational force of the unknown mass.
- Many electronic balances, or "scales" operate by same basic principle.
- Key is internal calibration of coil and magnet.

Basic Dimensions in the Current Metric System

International System of Units (SI)

Dimension	Symbol	SI Unit
Length	L	meter (m)
Mass	M	kilogram (kg)
Time	t	second (s)
Thermodynamic temperature	T	kelvin (K)
Electric current	I	ampere (A)
Amount of substance	?	mole (mol)
Luminous intensity	I_v	candela (cd)

- Choice of some of the basic units is somewhat arbitrary.
- Are all of these necessary?

Examples of Derived SI Units

Dimension	Symbol	SI Unit
Area	$A = L^2$	m^2
Volume	$V = L^3$	m^3
Velocity	$v = L/t$	m/s
Acceleration	$a = v/t = L/t^2$	m/s^2
Force	$f = ma = m \cdot L/t^2$	newton (N) = $kg \cdot m/s^2$
Energy	$E = f \cdot L$	joule (J) = Nm = $kg \cdot m^2/s^2$

Standard Prefixes for SI Units

prefix	abbreviation	multiplier	examples
nano	n	10^{-9}	nm, ng
micro	μ	10^{-6}	μm, μg
milli	m	10^{-3}	mm, mg
centi	c	10^{-2}	cm, cg
deci	d	10^{-1}	dm, dg
kilo	k	10^{3}	km, kg
mega	M	10^{6}	Mm, Mg

You should know these!

Using Units in Calculations

- Dimensional analysis:
  Treat dimensions and units as symbols that can be manipulated by standard rules of algebra.
- Conversion factors:
  - As a rule: “To convert from kilometers to meters, multiply by 1,000.”
  - As an equation: $1\,\text{km} = 1,000\,\text{m}$

Clicker Question #2

Which of these equations does *not* make sense:

A) $1\,\text{mg} = 1\,\text{g}/1,000$

B) $1\,\text{mile} = 1.609344\,\text{km}$

C) $\boxed{1\,\text{kg} = 1\,\text{liter}}$

D) $1\,\text{meter}/\text{s} = 1,1969\,\text{feet}/\text{hour}$

- To make sense, the **dimensions** on the two sides of the equations have to be consistent.

  The **units** do not have to be the same or be part of the same system.

Why isn't "$1\,\text{kg} = 1\,\text{liter}$" a Valid Equation?

- A true statement:
  1 L of water has a mass of 1 kg (within about 0.01 g at 4°C).
- Another (equivalent) true statement:
  The density of water, d_w, has a value of approximately 1 kg/L, or:

$$d_w = 1\,\text{kg}/\text{L}$$

- A valid equation to calculate the mass of a given volume of liquid:

$$\begin{aligned} \text{mass} &= \text{volume} \times \text{density} \\ &= 1\,\text{L} \times 1\,\text{kg}/\text{L} = 1\,\text{kg} \end{aligned}$$

- In mathematics, equality has a very specific meaning: Two things, after evaluation, are completely interchangeable.
- Mass and volume are not the same thing!

Algebraic Rearrangement of a Conversion Factor

$$1\,\text{km} = 1,000\,\text{m}$$

$$\frac{1\,\text{km}}{1,000\,\text{m}} = 1$$

$$\frac{1,000\,\text{m}}{1\,\text{km}} = 1$$

$$1\,\text{m} = 0.001\,\text{km}$$

- All of these equations are valid and equivalent.
- Any quantity can be multiplied (or divided) by 1 to give the same value.

Conversion by Multiplication

- Convert 37 miles to kilometers

$$1\,\text{mi} = 1.609344\,\text{km}$$
$$1.609344\,\text{km}/\text{mi} = 1$$

$$\begin{aligned} 37\,\text{mi} \times 1.609344\,\text{km}/\text{mi} &= 59.6\,\text{mi} \cdot \text{km}/\text{mi} \\ &= 59.6\,\text{km} \end{aligned}$$

- What if we divide instead of multiply?

$$37\,\text{mi} \div 1.609344\,\text{km}/\text{mi} = 23\,\text{mi}^2/\text{km}$$

This is correct algebraically, but it doesn't make much sense physically!

Clicker Question #3

"English" units for mass:

- 1 lb = 16 oz (avoirdupois)
- 1 oz = 28.349523125 g
- 1 stone = 14 Lb

If someone weighs 11 stone, what is that person's mass in kg?

A) $\sim 50\,\mathrm{kg}$

B) $\sim 70\,\mathrm{kg}$

C) $\sim 90\,\mathrm{kg}$

D) $\sim 110\,\mathrm{kg}$

Stones to kg

$$11\,\text{stone} \times 14\,\text{lb/stone} = 154\,\text{lb}$$

$$154\,\text{lb} \times 16\,\text{oz/lb} = 2.46 \times 10^3\,\text{oz}$$

$$2.46 \times 10^3\,\text{oz} \times 28.35\,\text{g/oz} = 6.98 \times 10^4\,\text{g}$$

$$6.98 \times 10^4\,\text{g} \div 1000\,\text{g/kg} \approx 70\,\text{kg}$$

$$1\,\text{kg} \approx 2.2\,\text{lb}$$

Clicker Question #4

How many hydrogen ions (H^+) are in a typical bacterium?

A) 1

B) 100

C) 1 thousand

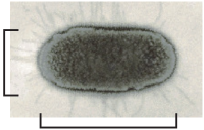
D) 1 million (10^6)

E) 1 billion (10^9)

All answers count (for now)!

Scale and Dimensions of a Bacterial Cell

- A typical bacterium found in the human gut: *Escherichia coli*



- Approximate this as a cylinder

- Volume of cylinder = $L \times A$

  L = length of cylinder

  A = area of cap = $\pi \times R^2$

Volume of Cylinder Representing a Bacterium

- Using units of μm:

$$
\begin{aligned}
V = L \times A = L \times \pi \times R^2 \\
&= 2\,\mu\text{m} \times \pi \times (0.5\,\mu\text{m})^2 \\
&= 1.6\,\mu\text{m}^3
\end{aligned}
$$

- Convert to m^3

$$1\,\mu\text{m} = 10^{-6}\,\text{m}$$

$$1.6\,\mu\text{m}^3 \times \left(10^{-6}\,\text{m}/\mu\text{m}\right)^3 = 1.6 \times 10^{-18}\,\text{m}^3$$

- What about liters or milliliters?

From Cubic Meters to Liters

- An easy-to-remember factoid: $1\,\text{cm}^3$ ("cc") $= 1\,\text{mL} = 10^{-3}\,\text{L}$
- And, $1\,\text{cm} = 10^{-2}\,\text{m}$

$$1\,\text{cm}^3 \times \left(10^{-2}\,\text{m}/\text{cm}\right)^3 = 10^{-6}\,\text{m}^3$$

$$1\,\text{mL} = 10^{-6}\,\text{m}^3$$

$$1\,\text{L} = 10^3\,\text{mL} \times 10^{-6}\,\text{m}^3/\text{mL} = 10^{-3}\,\text{m}^3, \qquad 1\,\text{m}^3 = 10^3\,\text{L}$$

- For our bacterium:

$$1.6 \times 10^{-18}\,\text{m}^3 \times 10^3\,\text{L}/\text{m}^3 = 1.6 \times 10^{-15}\,\text{L}$$

$$1.6 \times 10^{-15}\,\text{L} \times 10^3\,\text{mL}/\text{L} = 1.6 \times 10^{-12}\,\text{mL}$$

- How many bacteria would fit into 1 mL? In one 1 μL?