Physical Principles in Biology
Biology 3550
Spring 2025

Lecture 6:

Brownian Motion and the Plinko

Friday, 17 January 2025 ©David P. Goldenberg
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Announcements

- Problem set 1:
  - Due 11:59 PM, Friday, 24 January.
  - Download problems from Canvas.
  - Upload work to Gradescope.
  - Work must be typed!
- Quiz 1:
  - Friday, 24 January
  - 25 min, second half of class.
- Discussion/problem-solving sessions:
  - Wednesday, 22 January, 3:00 P.M. - CSC 13
  - Mondays (starting 27 January), 3:00 P.M. - AEB 306
- Office hours:
  - Wednesdays, 11:00 AM - ASB 306
  - Other times by appointment

Random Motion of Latex Beads in Water

Brownian Motion Movie

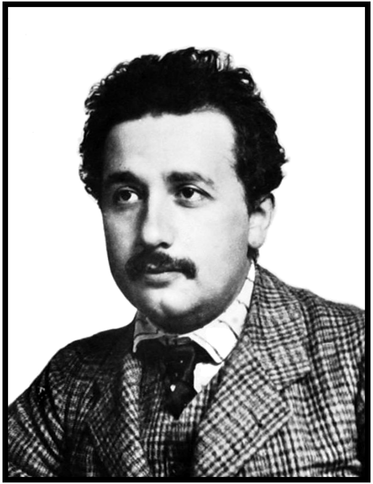
YouTube movie: https://www.youtube.com/watch?v=cDcprgWiQEY

Robert Brown

1773-1858

- Scottish botanist, explorer of Australia and exceptional microscopist.
- In 1827, observed random motions of small (1 μm) particles within pollen grains.
- Observed same motion in non-biological samples.
- What makes them move?

Albert Einstein



1879-1955 (photo 1904)

1905: Einstein's *Annus Mirabilis*

- Special relativity
- $E = mc^2$
- Photoelectric effect
- Brownian motion

Photograph by Lucien Chavan, https://en.wikipedia.org/wiki/Albert_Einstein

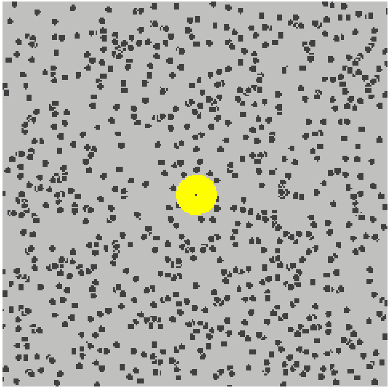
What About Mileva Marić?

Einstein and Marić

- Worked closely while both were university students (1896–1900), and after.
- Married in 1902; divorced in 1919.
- Some have argued that Marić made important contributions to the 1905 papers, especially special relativity.
- Very little historical record.

Esterson, A., Cassidy, D. C. & Sime, R. L. (2019). *Einstein's Wife: The Real Story of Mileva Einstein-Marić*. MIT Press. https://mitpress.mit.edu/books/einsteins-wife

Simulation of Brownian Motion



- Motion of particles is caused by fluctuations in the random motions of solvent molecules.
- Einstein made this model quantitative, allowing it to be rigorously tested.
- Conclusive evidence that liquids and gasses were made up of discrete molecules.
- A detailed, realistic molecular simulation is very difficult!

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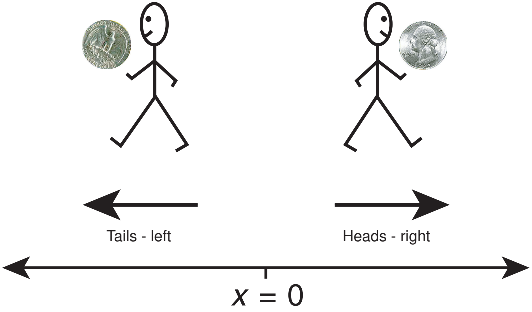
https://en.wikipedia.org/wiki/Brownian_motion
http://weelookang.blogspot.com/2010/06/ejs-open-source-brownian-motion-gas.html

Displacement from the Starting Point for a One-Dimensional Random Walk



- Start at position $x = 0$.
- Take n random steps to the right or left.

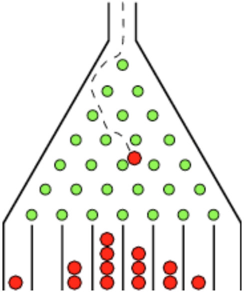
  $n_{\mathrm{H}} =$ no. of heads

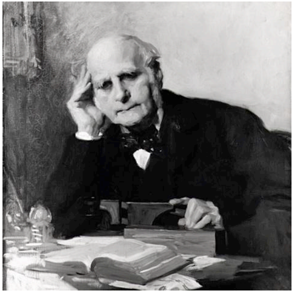
  $n_{\mathrm{T}} =$ no. of tails
- Final position is x.

  $$x = n_{\mathrm{H}} - n_{\mathrm{T}}$$

  (in units of step lengths)
- Generally expect a distribution of x if the random walk is repeated a large number of times.

Analog of a Random Walk: The Galton Probability Machine





- Sir Francis Galton (1822-1911)
  - Cousin of Charles Darwin.
  - Attempted to find a mathematical description of genetic variation and evolution.
  - Early advocate of eugenics (invented the term); improvement of humans by selective breeding.

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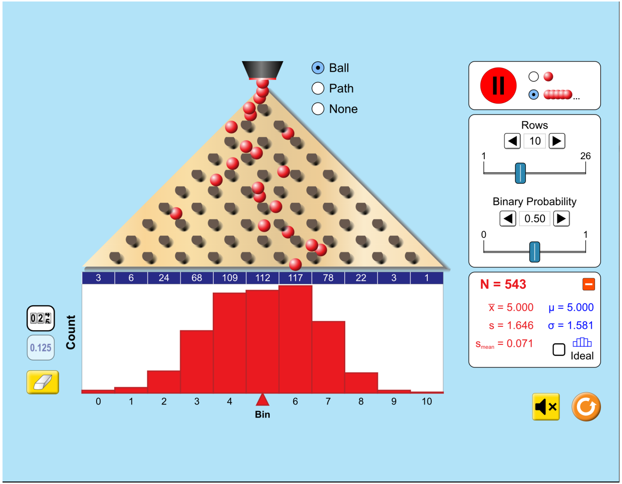
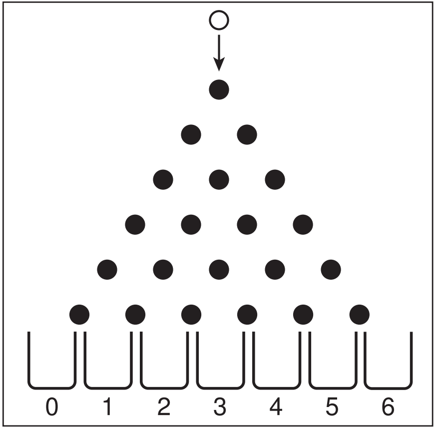
http://mathworld.wolfram.com/GaltonBoard.html
https://en.wikipedia.org/wiki/Francis_Galton

Plinko Computer Demonstration



https://phet.colorado.edu/en/simulation/plinko-probability

Plinko Probabilities: A Six-row Plinko



- The question: What is the probability that a ball will fall in each of the buckets.
- For N plinko rows, there will be $N + 1$ buckets for balls to land in.
- For convenience, buckets are numbered from 0 to N.
- How shall we define the sample set?

Clicker Question #1

How shall we define the plinko sample set?

A) By the number of the bin that the ball falls into.

B) By the individual paths that the balls can follow.

C) By whether the ball falls on the right side, the left side or the middle.

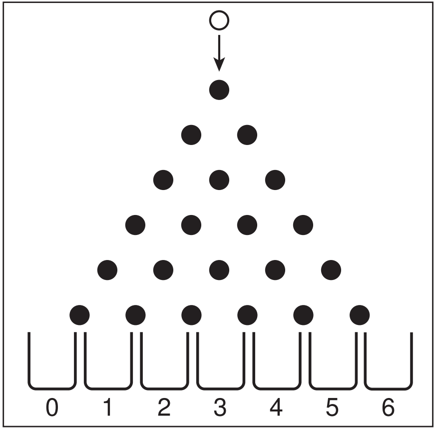
D) By how far the ball falls from the center.

All answers count for now.

Count The Paths to Reach a Given Bucket

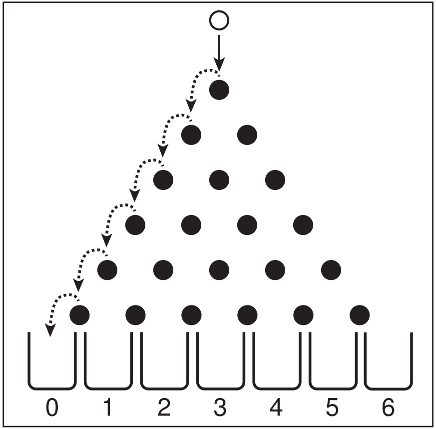
- Define outcomes as all of the possible paths, because they all have the same probability (if the plinko is unbiased).
- Define events as final positions of ball, *i.e.,* bucket numbers.
- How many possible paths are there?
  - For a 6-row plinko, each path involves 6 places to change direction.
  - The number of different paths is: $2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^6 = 64$
  - Each path has an equal probability, equal to 1/64
  - For an n-row plinko, the number of different paths is 2^n, and the probability of each is $1/2^n = 2^{-n}$.

Need to Count the Number of Paths to Each Final Position



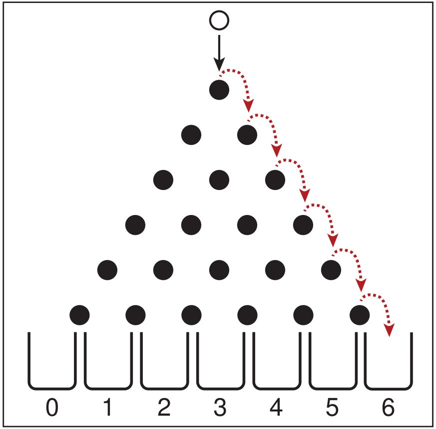
Bucket No.	Paths
0	
1	
2	
3	
4	
5	
6	

How Many Paths to Bucket 0?



Bucket No.	Paths
0	1
1	
2	
3	
4	
5	
6	

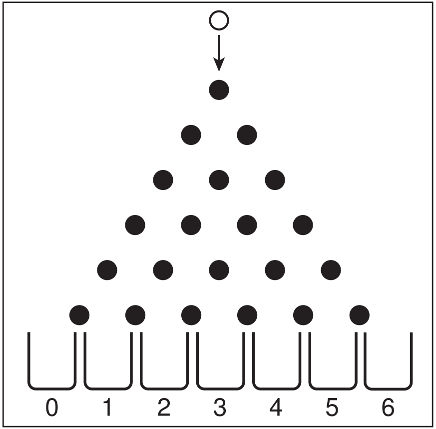
How Many Paths to Bucket 6?



Bucket No.	Paths
0	1
1	
2	
3	
4	
5	
6	1

Clicker Question #2

How many paths are there to bucket 1?



A) 2

B) 3

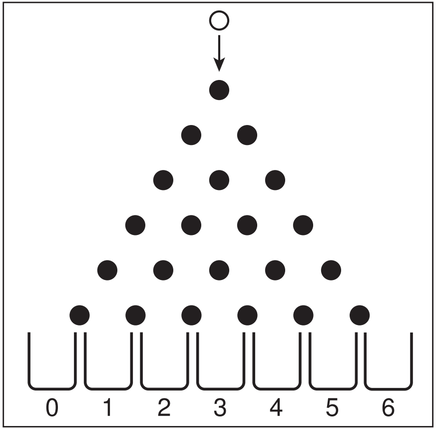
C) 4

D) 5

E) 6

All answers count for now.

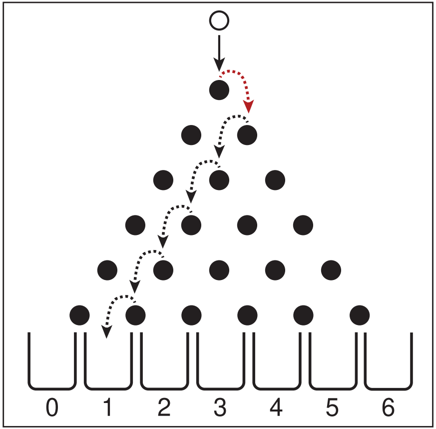
How Many Paths to Bucket 1?



Bucket No.	Paths
0	1
1	
2	
3	
4	
5	
6	1

- Each path to bucket 1 includes 1 turn to the right and 5 to the left.
- There are 6 places to turn right.

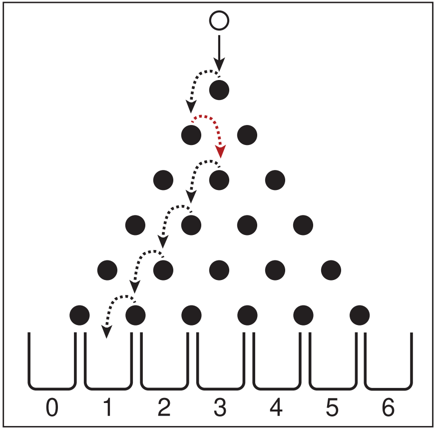
How Many Paths to Bucket 1?



Bucket No.	Paths
0	1
1	
2	
3	
4	
5	
6	1

- Each path to bucket 1 includes 1 turn to the right and 5 to the left.
- There are 6 places to turn right.

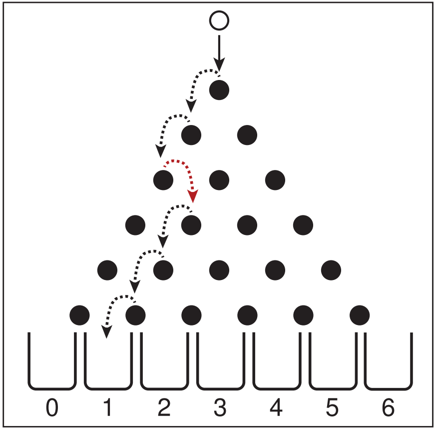
How Many Paths to Bucket 1?



Bucket No.	Paths
0	1
1	
2	
3	
4	
5	
6	1

- Each path to bucket 1 includes 1 turn to the right and 5 to the left.
- There are 6 places to turn right.

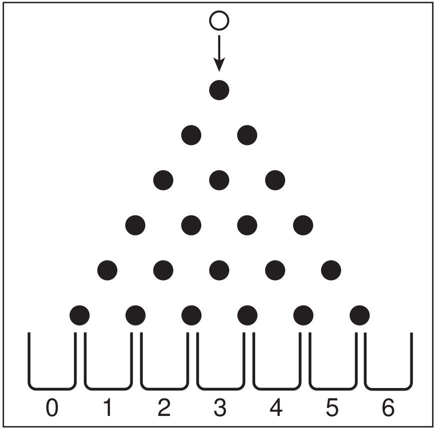
How Many Paths to Bucket 1?



Bucket No.	Paths
0	1
1	6
2	
3	
4	
5	
6	1

- Each path to bucket 1 includes 1 turn to the right and 5 to the left.
- There are 6 places to turn right.

How Many Paths to Bucket 5?

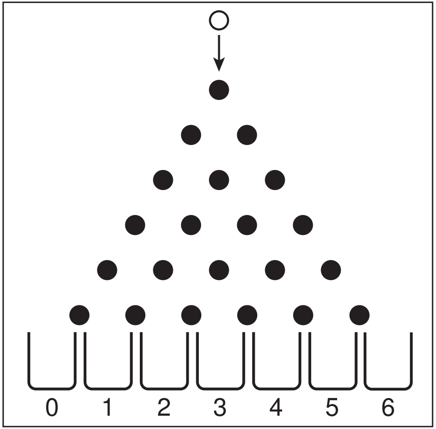


Bucket No.	Paths
0	1
1	6
2	
3	
4	
5	6
6	1

- Each path to bucket 5 includes 5 turns to the right and 1 to the left.

Clicker Question #3

How many paths are there to bucket 2?



A) 7

B) 8

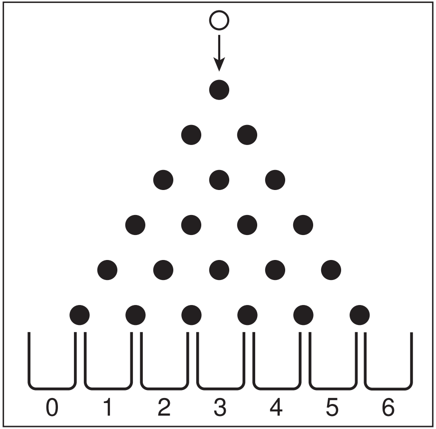
C) 12

D) 15

E) 20

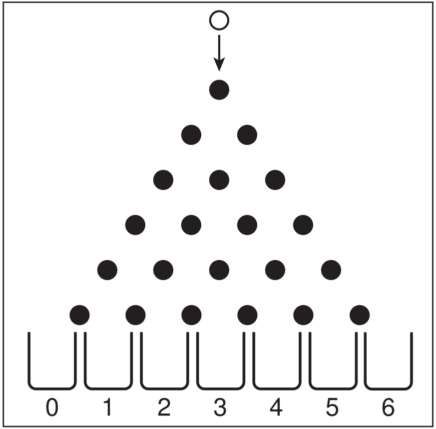
All answers count for now.

How Many Paths to Bucket 2?



Bucket No.	Paths
0	1
1	6
2	
3	
4	
5	6
6	1

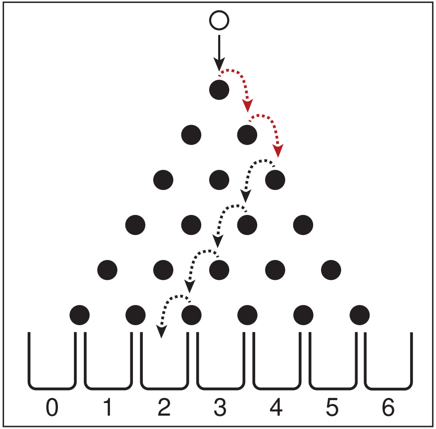
How Many Paths to Bucket 2?



Bucket No.	Paths
0	1
1	6
2	
3	
4	
5	6
6	1

- Each path to bucket 2 includes 2 turns to the right and 4 to the left.

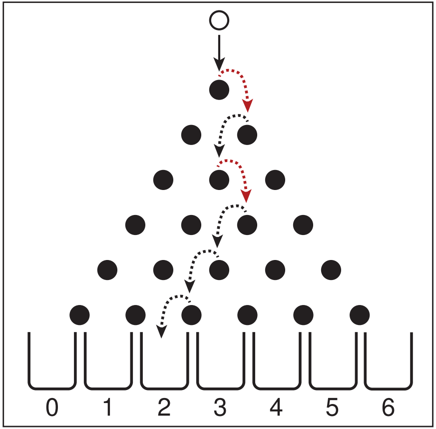
How Many Paths to Bucket 2?



Bucket No.	Paths
0	1
1	6
2	
3	
4	
5	6
6	1

- Each path to bucket 2 includes 2 turns to the right and 4 to the left.

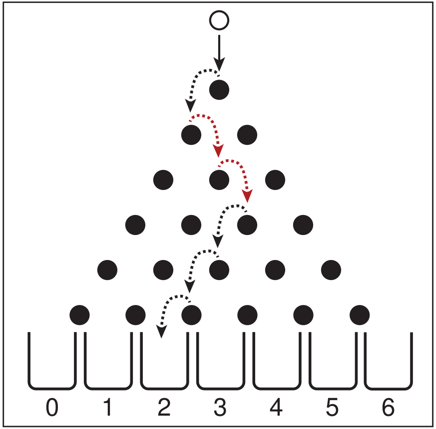
How Many Paths to Bucket 2?



Bucket No.	Paths
0	1
1	6
2	
3	
4	
5	6
6	1

- Each path to bucket 2 includes 2 turns to the right and 4 to the left.

How Many Paths to Bucket 2?

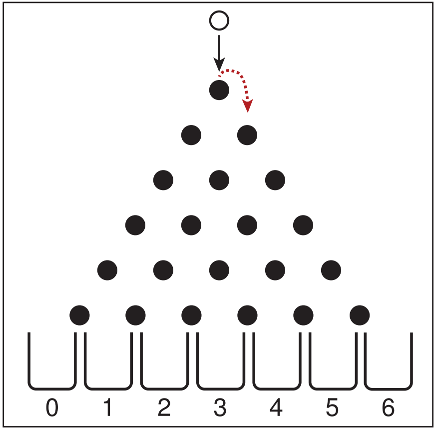


Bucket No.	Paths
0	1
1	6
2	
3	
4	
5	6
6	1

- Each path to bucket 2 includes 2 turns to the right and 4 to the left.

How Many Paths to Bucket 2?

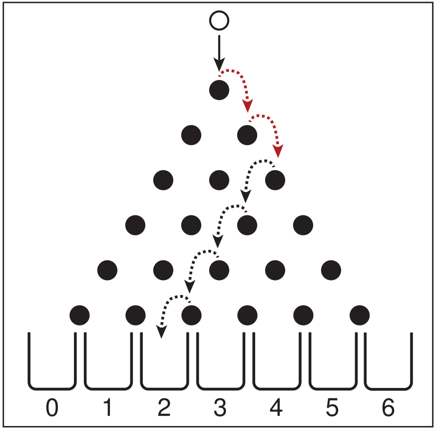
If the first turn is to the right, how many paths are there to bucket 2?



1st right turn row	Paths
1	
2	
3	
4	
5	
6	

How Many Paths to Bucket 2?

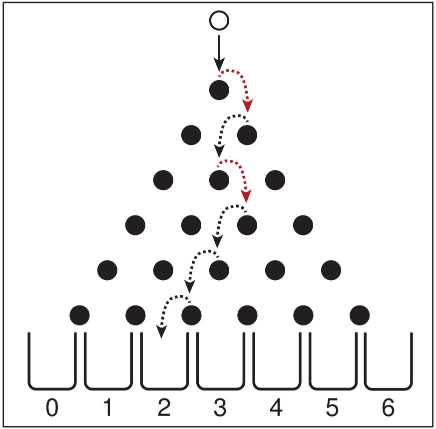
If the first turn is to the right, how many paths are there to bucket 2?



1st right turn row	Paths
1	
2	
3	
4	
5	
6	

How Many Paths to Bucket 2?

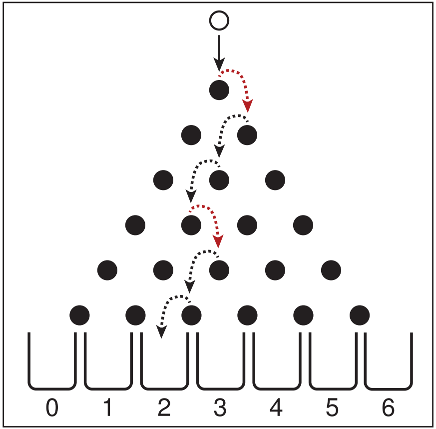
If the first turn is to the right, how many paths are there to bucket 2?



1st right turn row	Paths
1	
2	
3	
4	
5	
6	

How Many Paths to Bucket 2?

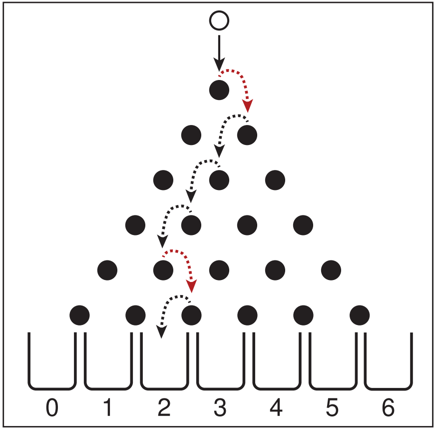
If the first turn is to the right, how many paths are there to bucket 2?



1st right turn row	Paths
1	
2	
3	
4	
5	
6	

How Many Paths to Bucket 2?

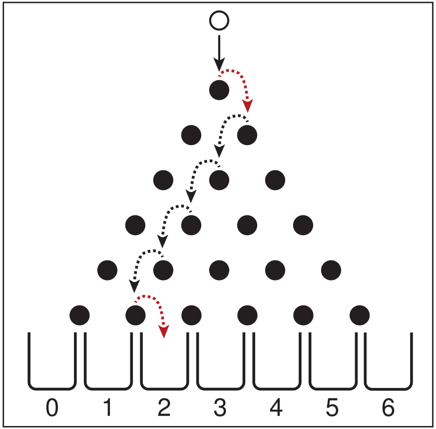
If the first turn is to the right, how many paths are there to bucket 2?



1^{st} right turn row	Paths
1	
2	
3	
4	
5	
6	

How Many Paths to Bucket 2?

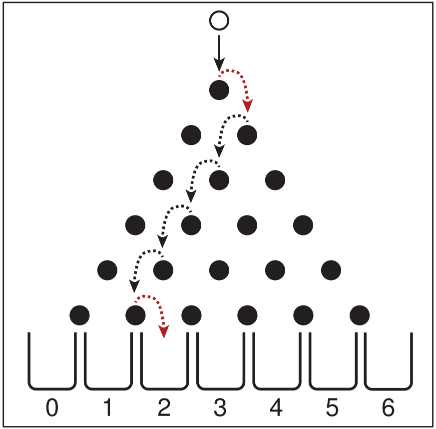
If the first turn is to the right, how many paths are there to bucket 2?



1^{st} right turn row	Paths
1	
2	
3	
4	
5	
6	

How Many Paths to Bucket 2?

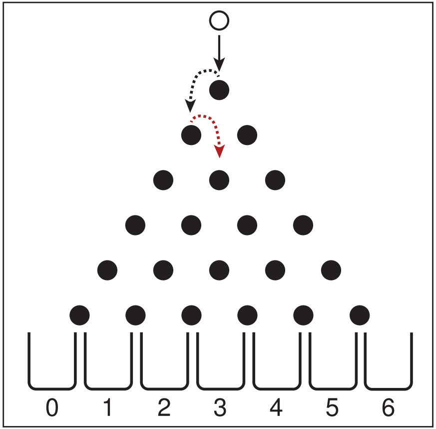
If the first turn is to the right, how many paths are there to bucket 2?



1^{st} right turn row	Paths
1	5
2	
3	
4	
5	
6	

How Many Paths to Bucket 2?

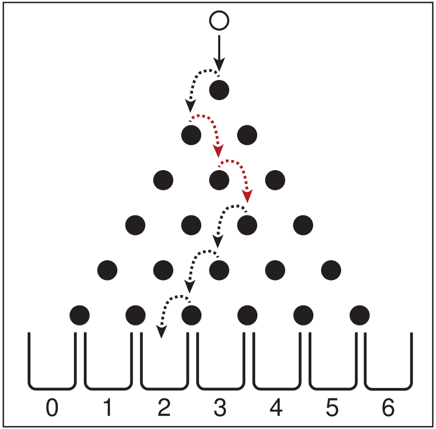
If the first turn to the right is at row 2, how many paths to bucket 2?



1st right turn row	Paths
1	5
2	
3	
4	
5	
6	

How Many Paths to Bucket 2?

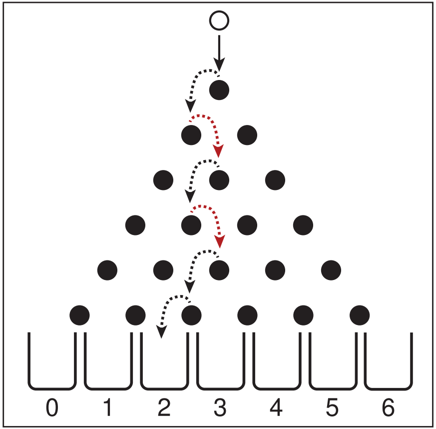
If the first turn to the right is at row 2, how many paths to bucket 2?



1st right turn row	Paths
1	5
2	
3	
4	
5	
6	

How Many Paths to Bucket 2?

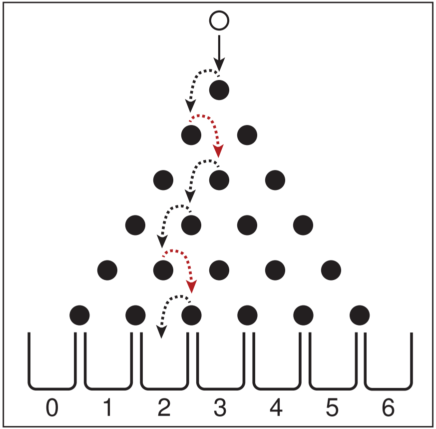
If the first turn to the right is at row 2, how many paths to bucket 2?



1st right turn row	Paths
1	5
2	
3	
4	
5	
6	

How Many Paths to Bucket 2?

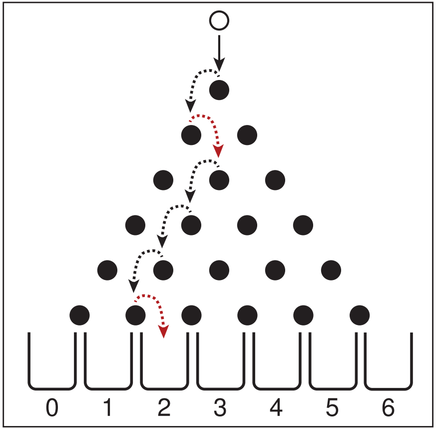
If the first turn to the right is at row 2, how many paths to bucket 2?



1^{st} right turn row	Paths
1	5
2	
3	
4	
5	
6	

How Many Paths to Bucket 2?

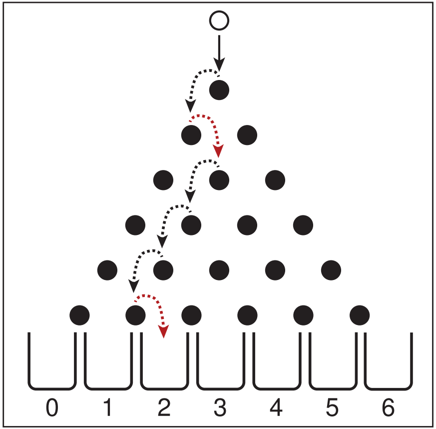
If the first turn to the right is at row 2, how many paths to bucket 2?



1^{st} right turn row	Paths
1	5
2	
3	
4	
5	
6	

How Many Paths to Bucket 2?

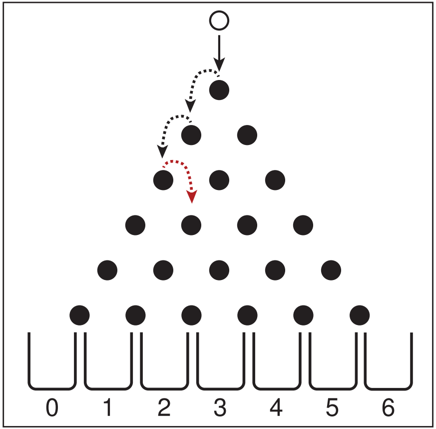
If the first turn to the right is at row 2, how many paths to bucket 2?



1^{st} right turn row	Paths
1	5
2	4
3	
4	
5	
6	

How Many Paths to Bucket 2?

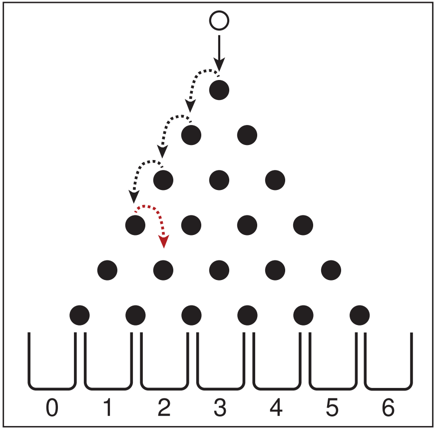
If the first turn to the right is at row 3, how many paths to bucket 2?



1^{st} right turn row	Paths
1	5
2	4
3	3
4	
5	
6	

How Many Paths to Bucket 2?

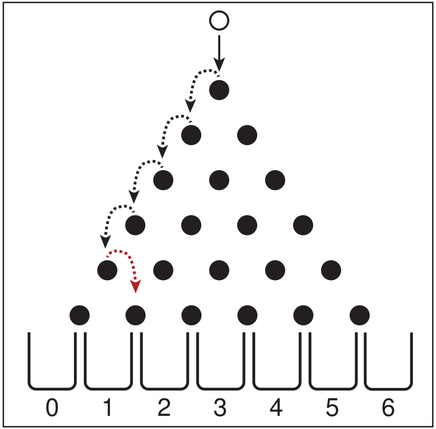
If the first turn to the right is at row 4, how many paths to bucket 2?



1^{st} right turn row	Paths
1	5
2	4
3	3
4	2
5	
6	

How Many Paths to Bucket 2?

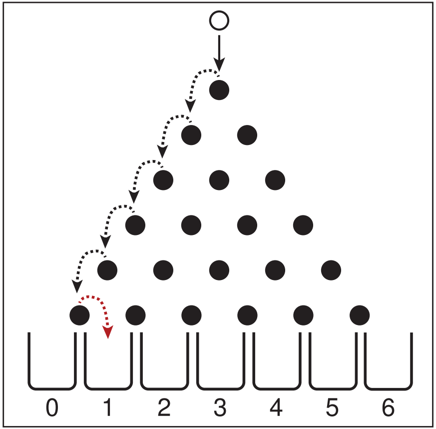
If the first turn to the right is at row 5, how many paths to bucket 2?



1^{st} right turn row	Paths
1	5
2	4
3	3
4	2
5	1
6	

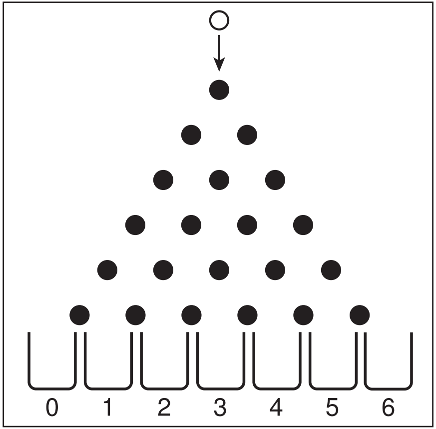
How Many Paths to Bucket 2?

If the first turn to the right is at row 6, how many paths to bucket 2?



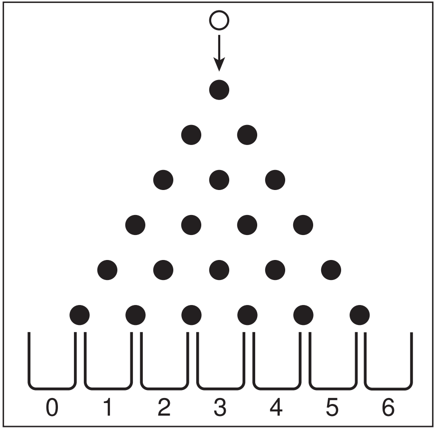
1^{st} right turn row	Paths
1	5
2	4
3	3
4	2
5	1
6	0

How Many Paths to Bucket 2?



Bucket No.	Paths
0	1
1	6
2	15
3	
4	
5	6
6	1

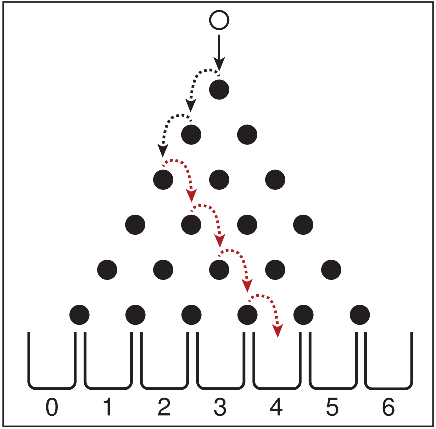
How Many Paths to Bucket 4?



Bucket No.	Paths
0	1
1	6
2	15
3	
4	
5	6
6	1

- Each path to bucket 4 includes 4 turns to the right and 2 to the left.

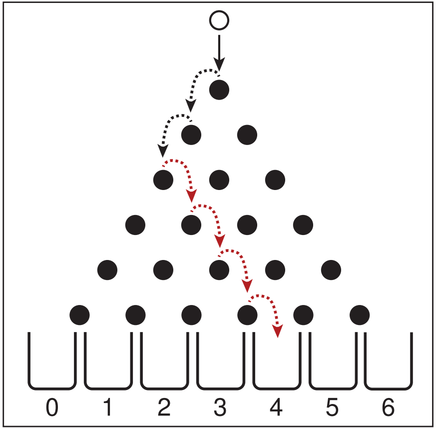
How Many Paths to Bucket 4?



Bucket No.	Paths
0	1
1	6
2	15
3	
4	
5	6
6	1

- Each path to bucket 4 includes 4 turns to the right and 2 to the left.

How Many Paths to Bucket 4?

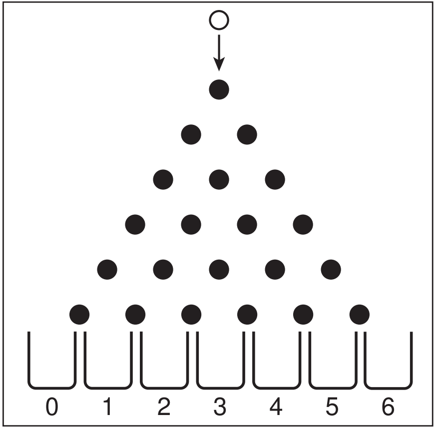


Bucket No.	Paths
0	1
1	6
2	15
3	
4	15
5	6
6	1

- Each path to bucket 4 includes 4 turns to the right and 2 to the left.

Clicker Question #4

How many paths are there to bucket 3?



A) $\boxed{20}$

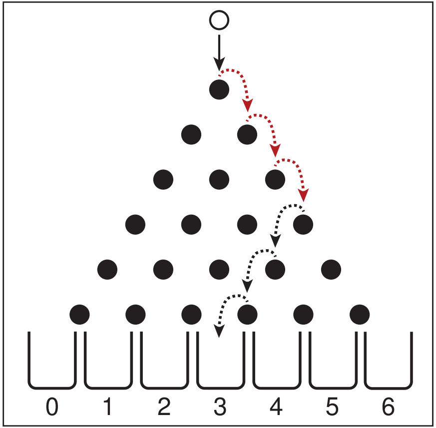
B) 25

C) 30

D) 35

E) 40

How Many Paths to Bucket 3



Bucket No.	Paths
0	1
1	6
2	15
3	20
4	15
5	6
6	1

- The total number of paths is 64.
- Counting the paths to bucket 3 looks hard!

Another Way to Count the Paths to Bucket 2

- 2 turns to the right and 4 turns to the left.
- There are 6 rows where one turn could be placed.
- There are 5 rows where a second turn could be placed.
  $6 \times 5 = 30$
- BUT, this assumes that the turns can be placed in either order!
- The first turn has to come before the second, so each possible path has been counted twice.
- The correct count: $6 \times 5 \times \frac{1}{2} = 15$
- A general strategy: Count all of the possible places where each right turn could be placed, allowing all possible orders, and then correct for over counting.

Counting the Paths to Bucket 3

- 3 turns to the right and 3 to the left.
- Ignoring the order of placement:
  - 6 rows where a first right turn can be placed.
  - 5 rows where a second right turn can be placed.
  - 4 rows where a third right turn can be placed.
  - $6 \times 5 \times 4 = 120$
- But, this assumes turns can be placed in any order!
- How do we determine how many times the paths have been over counted?