Physical Principles in Biology
Biology 3550
Fall 2024

Lecture 8:

A Biased Plinko and the Binomial Distribution

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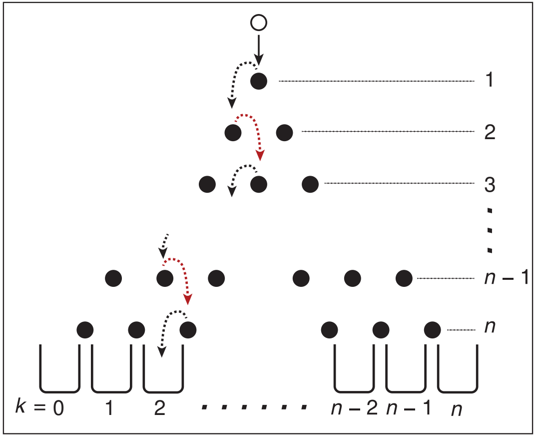
An n-row Plinko



- k = bucket number.
- To reach bucket k, ball must make k turns to the right and $n - k$ turns to the left.

The Universal Plinko Formula

- For an n-row plinko, the number of paths to bucket k is the number of ways to place k labeled beans in n cups **in a single order**.
- The number of ways to put k labeled beans in n cups, **in any order**, is:

$$n(n-1)(n-2)\cdots(n-k+1) = \frac{n!}{(n-k)!}$$

- The number of ways to place k labeled beans in k cups **in any order** is:

$$\frac{k!}{(k-k)!} = k!$$

- The number of ways to place k labeled beans in n cups **in a single order** is:

$$\frac{n!}{k!(n-k)!}$$

Probabilities for an n-row Plinko

- The total number of paths is 2^n.
- If each turn to the right or left is equally probable, the probabilities of all paths are equal, and the probability of each path is:

$$p = \left(\frac{1}{2}\right)^n = \frac{1}{2^n} = 2^{-n}$$

- The probability of a ball landing in bucket k is the number of paths to the bucket multiplied by the probability of each path:

$$p(k) = \frac{n!}{k!(n-k)!} \cdot 2^{-n}$$

What if the Plinko is Biased?

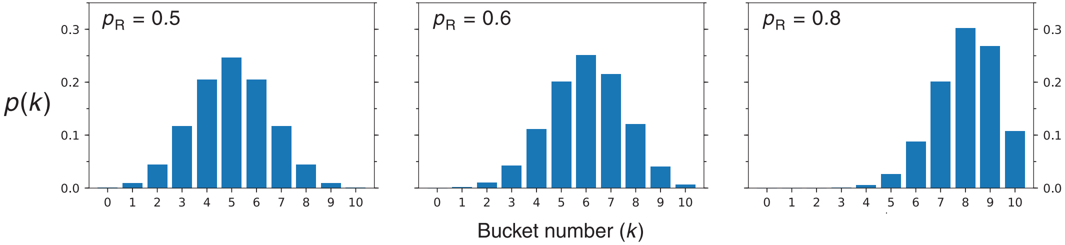
- Suppose that each peg in the plinko has been “fixed”, so that the probability of a left turn is 0.4 and the probability of a right turn is 0.6.
- For each of the paths to bucket k, there are k right turns and $(n-k)$ left turns.
- For each individual path to bucket k, the probability is:

$$0.6^k \times 0.4^{(n-k)}$$

- The total probability of a ball falling in bucket k is:

$$p(k) = \frac{n!}{k!(n-k)!} \times 0.6^k \times 0.4^{(n-k)}$$

Probabilities for a Biased 10-row Plinko



- Biased pegs "push" balls to the right.
- Probability (number of paths) "draws" balls to the center.
- Can you think of physical processes like this?

The Binomial Probability Distribution Function

- The general formulation:
  $p(k; n, p_s)$ is the probability of k successes in n successive binary (yes/no) trials when the probability of success in each trial is p_s.
- The probability function:

$$p(k; n, p_s) = \frac{n!}{k!(n-k)!} p_s^k (1 - p_s)^{(n-k)}$$

  for $k = 0$ to n.
- Some applications beyond plinkos:
  - Number of heads in n successive coin tosses.
  - Number of successes in prescribing a medication to a series of patients with the same condition.
  - Probability of surviving n potentially deadly events.
    $p(n; n, p_s)$, were p_s is the probability of surviving each event.
    (Only counts for n successes!)